ON CERTAIN PROPERTY OF THE NORMS BY MODULARS

By

Tetsuya SHIMOGAKI

Let R be a universally continuous semi-ordered linear space. A functional $m(a)(a\in R)$ is said to be a modular[1] on R if it satisfies the following modular conditions:

(1) $0\leqq m(a)\leqq\infty$ for all $a\in R$;

(2) if $m(\xi a)=0$ for all $\xi>0$, then $a=0$;

(3) for any $a\in R$ there exists $\alpha>0$ such that $m(\alpha a)<\infty$;

(4) for every $a\in R$, $m(\xi a)$ is a convex function of ξ;

(5) $|a|\leqq|b|$ implies $m(a)\leqq m(b)$;

(6) $a\frown b=0$ implies $m(a+b)=m(a)+m(b)$;

(7) $0\leqq a_\lambda\uparrow_{\lambda\in\Lambda}a$ implies $m(a)=\sup\limits_{\lambda\in\Lambda}m(a_\lambda)$.

In R, we define functionals $\|a\|$, $|||a|||$ $(a\in R)$ as follows

$$\|a\|=\inf_{\xi>0}\frac{1+m(\xi a)}{\xi}, \quad |||a|||=\inf_{m(\xi a)\leqq 1}\frac{1}{|\xi|}.$$

Then it is easily seen that both $\|a\|$ and $|||a|||$ are norms on R and $|||a|||\leqq\|a\|\leqq 2|||a|||$ for all $a\in R$. $\|a\|$ is said to be the first norm by m and $|||a|||$ is said to be the second norm by m. Let $\bar{R}^m$ be the modular conjugate space of R and $\bar{m}$ be the conjugate modular of m[2] then we can introduce the norms by $\bar{m}$ as above. It is known that if R is semi-regular, the first norm by the conjugate modular $\bar{m}$ is the conjugate norm of the second norm by m and the second norm by the conjugate modular $\bar{m}$ is the conjugate norm of the first norm by m. Since $\|a\|$ and $|||a|||$ are semi-continuous by (7), they are reflexive norms (cf. [7]).

If a modular m is of L_p-type, i.e., $m(\xi x)=\xi^p m(x)$ for all $x\in R$, $\xi\geqq 0$,

1) We owe the notations and the terminologies using here to the book: H. NAKANO [3].

2) The conjugate modular $\bar{m}$ is defined as $\bar{m}(\bar{a})=\sup\limits_{x\in R}\{\bar{a}(x)-m(x)\}$ for every $\bar{a}\in\bar{R}^m$, where $\bar{R}^m$ is the space of the modular bounded universally continuous linear functionals on R.