PARTIALLY ORDERED ABELIAN SEMIGROUPS. IV

ON THE EXTENTION OF THE CERTAIN NORMAL PARTIAL ORDER DEFINED ON ABELIAN SEMIGROUPS

By

Osamu NAKADA

In Part I[1] of this series, I noted that for any two elements x and y non-comparable in the strong partial order P defined on an abelian semigroup S there exists an extension Q of P such $x>y$ in Q if and only if P is normal. In this Part IV, I shall discuss the extension of the partial order under the weak condition than strongness.

Definition 1. A set S is said to be *a partially ordered abelian semigroup* (p.o. semigroup), when S is (I) an abelian semigroup (not necessarily contains the unit element), (II) a partially ordered set, and satisfies (III) the homogeneity: $a \geqq b$ implies $ac \geqq bc$ for any c of S.

A partial order which satisfies the condition (III) is called a *partial order defined on an abelian semigroup.*

Moreover, if a partial order defined on an abelian semigroup S is a linear order, then S is said to be a *linearly ordered abelian semigroup* (l.o. semigroup).

We write $a // b$ in P for a and b are non-comparable in P.

Definition 2. Let P be a partial order defined on an abelian semigroup S. We consider the following conditions for the partial order P:

(E): $ac>bc$ in P implies $a>b$ in P. (order cancellation law)

(G): Let x and y be any two elements non-comparable in P. Then there exists an extension of P in which $x>y$.

(H): If $a // b$ in P, then $ua \neq ub$ for any u in S.

(K): If $a // b$ in P, then $ua // ub$ in P for any u in S.

(L): Let $a // b$ and $u // v$ in P respectively. If $au \neq bv$, then $au // bv$ in P.

1) Partically ordered abelian semigroup. I. On the extension of the strong partial order defined on abelian semigroups. Journ. Fac. Sci., Hokkaido University, Series I, vol. XI (1951), pp. 181–189.