ON THE GIBBS PHENOMENON

By

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1. Introduction.

The Gibbs phenomenon of the Fourier series $\sum_{n=1}^{\infty}\frac{\sin nx}{n}$ was early found by W. Gibbs [1], see A. Zygmund [22]. And further H. Cramér [2], T. H. Grownall [3], B. Kuttner [4] [5] [6], O. Szász [9] [10] [11], M. Cheng [12], L. Lorch [13] L. Ching-Hsi [18], A. E. Livingston [19] and the author [20] investigated the Gibbs phenomenon of the same Fourier series for various kinds of means: Cesàro, Riesz, Euler, etc..

The Gibbs phenomenon of the Fourier series of a function which has a discontinuity point of the second kind was recently investigated by S. Izumi and M. Satô [14] [15] and B. Kuttner [7] [8]. The author [16] [17] proved some theorems concerning the Gibbs phenomenon of the Fourier series of this kind for Cesàro means. The object of the present paper is to study the Gibbs phenomenon of such Fourier series for Riesz, Borel, Euler and Hausdorff means.

2. B. Kuttner [6] proved the following

Theorem 1. *If $0<\lambda<2$, there is a function $r(\lambda)$ such that the Gibbs phenomenon vanishes for the means (R, n^{λ}, κ) of the Fourier series of a function having a simple discontinuity if $\kappa\geqq r(\lambda)$, but not if $\kappa<r(\lambda)$. The function $r(\lambda)$ is continuous and (strictly) increasing, and is, for all $\lambda<2$, less than the function $k(\lambda)$ defined in [5]. It tends to 0 as $\lambda\to 0$, equals Cramér's constant r_0 when $r=1$, see [2], and tends to infinity as $\lambda\to 2$. If $\lambda=2$, the Gibbs phenomenon persists for the means (R, n^{λ}, κ) however large κ may be.*

We shall extend this theorem to the discontinuity point of the second kind satisfying the following conditions, see [14] [15]. That is

Theorem 2. *Let $f(x)$ be an odd function about ξ, and suppose that*

$$f(x)=l\psi(x-\xi)+g(x),$$

where $\psi(x)$ is a periodic function with period 2π such that

$$\psi(x)=(\pi-x)/2 \qquad (0<x<2\pi),$$

and where

$$\limsup_{x\downarrow\xi} f(x)=l\pi/2, \qquad \liminf_{x\uparrow\xi} f(x)=-l\pi/2,$$