CONTINUOUS FILTERING AND ITS SPECTRAL SEQUENCE

By

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0. A filtering f of a ring A is a integer valued function on A satisfying the following three conditions:

$$f(x+y) \geq \min\{f(x), f(y)\}, \qquad x, y \in A, \tag{0.1}$$

$$f(xy) \geq f(x) + f(y), \tag{0.2}$$

$$f(0) = +\infty. \tag{0.3}$$

Thus, the notion of filtering can be regarded as a generalization of discrete valuation of a field. For purely algebraic interest, it seems to be natural to consider a continuous filtering as the generalization of continuous valuation.

In this note, we consider a real valued function F on A satisfying the above three conditions. We call F a *continuous filtering* of A, and the ring A is said to be a *continuously filtered ring*.

Sections 1 and 2 are devoted to describe analogous definitions notations and relations to those of J. Leray [1], and the main parts of this note are sections 3 and 4.

1. A ring A is called a *continuosly graded ring* if

$$A = \sum_{p \in R} A^{[p]} \qquad \text{(direct sum, } R \text{ is the set of reals)}$$

where $\{A^{[p]}\}$ are submodules of A and satisfy

$$A^{[p]} \cdot A^{[q]} \subset A^{[p+q]}.$$

A continuously filtered ring A is called a *continuously filtered differential ring* if A has a differentiation (d, a) subjected to

$$d^2 = 0,$$

$$adx + dax = 0, \qquad x, y \in A,$$

$$d(xy) = dx \cdot y + ax \cdot dy, \qquad (a \text{ is an automorphism of } A),$$

and

$$F(ax) = F(x).$$

A differentiation (d, a) is called homogeneous of degree r $(r \in R)$ if