VECTOR FIELDS AND SPACE FORMS

By

Toshiyuki MAEBASHI

The surface of rotation R in an $(n+1)$-dimensional Euclidean space has some remarkable intrinsic properties. Among them, the following two are the most typical:

Property I. Let G be the Lie group consisting of all isometries. Then there exists a subgroup H of G of the type:

$$\exists x \in R\,[\dim H(x) = n-1],$$

provided that $H(x)$ denotes the orbit of x.

Property II. There exists a vector field $\boldsymbol{V}$ which either is parallel or satisfies these three conditions:

(i) In case of the movement in the direction orthogonal to $\boldsymbol{V}$, the end point of $\boldsymbol{V}$ is always fixed (intrinsically, with respect to Levi-Civita parallelism of R).

(ii) The trajectories of $\boldsymbol{V}$ are geodesics in regard to the induced metric of R.

(iii) $\boldsymbol{V}$ admits a family of transversal hypersurfaces.

Remarks. It is worth noting that from the global point of view the above mentioned vector field $\boldsymbol{V}$ may generally have certain singularities,[1] particularly in case of R being a closed hypersurface.

Property I and II give rise conversely to the interesting questions of determing the global nature of Riemannian spaces possessing either Property I or Property II respectively. Each of these questions propounds quite a different problem than the other and the methods by means of which these problems can be solved must differ very much from each other. In either case, however, results to be obtained will show that the spaces in question have remarkable similarity to the surface of rotation. Conversely speaking, to solve these problems is in a sense nothing but to make clear this similarity which such spaces have.

In fact it is from this point of view that P. Mostert has dealt with the former problem and determined it to a large extent [1]. It must

1) The word singularity means that $\boldsymbol{V}$ may have not only 0-points, but also a kind of discontinuity.