ON THE FINITENESS OF MODULARED SPACES

By

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Let R be *a modulared space*[1] with the modular $m(x)$[2] $(x \in R)$. R or m is said to be *finite* if $m(x) < \infty$ for every $x \in R$.

Since the finite modulars are rather convenient to be treated, they were studied from earlier steps of investigation on these spaces by some authors.

W. Orlicz and Z. Birnbaum [6] found a necessary and sufficient condition (so-called Δ_2-condition) in order that Orlicz spaces are finite. After that, this fact was generalized for an arbitrary monotone complete[3] modular by means of finding a formula which characterizes the finite modular, on non-atomic and atomic spaces by I. Amemiya [1] and by T. Shimogaki [7] respectively.

On the other hand, H. Nakano [3] defined *a modulared fuction space*, a kind of the generalizations of the Orlicz spaces, and showed that an arbitrary modulared space could be represented by a modulared function space. Hence, the modulared function space may be considered as the most general space among the concrete examples of the modulared space.

In this paper, we shall first characterize the finite modular by its another formula in the both cases of non-atomic (§ 1) and atomic (§ 2) spaces. Next, relating to the finiteness of the modular, we consider the continuity of the modular norms (§ 3). However, our main purpose lies in the application of the formula in question to the modulared function space to get a generalization of Orlicz-Birnbaum's Δ_2-condition (§ 4). Moreover, we discuss the conjugate property of the finite modular in connection with those characterizations (§ 5).

Unless otherwise stated, m is always monotone complete throughout this paper. And that is not too restrictive, because our problems are ultimately concerned with the modulared function space.

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1) The modulared space is defined by H. Nakano [3] and have been studied mainly by him and his school.

2) Terminologies and notations in this paper are also due to [3].

3) m is said to be monotone complete if $0 \leqq x_\lambda \uparrow_{\lambda \in \Lambda}$ with $\sup_{\lambda \in \Lambda} m(x_\lambda) < \infty$ implies the existence of $\bigcup_{\lambda \in \Lambda} x_\lambda$.