MONOTONY AND FLATNESS OF THE NORMS BY MODULARS

By

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§ 1. Introduction

Let R be a modulared semi-ordered linear space with a modular m[1]. We define the two norms on R by the formulas:

(a) $$\|x\| = \inf_{\xi>0} \frac{1+m(\xi x)}{\xi},$$

(b) $$|||x||| = \inf_{m(\xi x)\leqq 1} \frac{1}{|\xi|}.$$

The norms $\|x\|$ and $|||x|||$ are called the first norm and the second norm by m respectively.

$\tilde{R}$ denotes the totality of all linear functionals which are bounded under the norm $\|x\|$. The associated modular $\tilde{m}$ of m is defined on $\tilde{R}$ by the formula:

(c) $$\tilde{m}(\tilde{x}) = \sup_{x\in R} \{\tilde{x}(x) - m(x)\} \qquad \text{for all } \tilde{x}\in\tilde{R}.$$

The functional $\tilde{m}$ satisfies all the modular conditions (cf. [2, §38]).

We know (cf. [3, §§80–83])

(d) $$m(a) = \sup_{\tilde{x}\in\tilde{R}} \{\tilde{x}(a) - \tilde{m}(\tilde{x})\} \qquad \text{for all } a\in R,$$

and

(e) $$\|a\| = \sup_{\tilde{m}(\tilde{x})\leqq 1} |\tilde{x}(a)| \qquad \text{for all } a\in R.$$

The first and second norms satisfy always

(f) $$|||x||| \leqq \|x\| \leqq 2|||x||| \qquad \text{for all } x\in R,$$

and a fortiori, they are equivalent (cf. [3, §83]). The first and second norms by the associated modular $\tilde{m}$ on $\tilde{R}$ are denoted by $\|\tilde{x}\|$ and $|||\tilde{x}|||$ respectively. Then we know ([3, §83]),

1) We use definitions, notation and terminology of [2,3].