

CONVEXITY AND EVENNESS IN MODULARED SEMI-ORDERED LINEAR SPACES

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CONTENTS

Introduction	59
Chapter I. Convexity and evenness of the norms	63
§1. Preliminary lemmas	63
§2. Strict convexity and evenness of the norms	68
§3. Uniform convexity and uniform evenness of the norms	72
Chapter II. Equivalent modulars	76
§4. Some topological properties	76
§5. Methods of construction of new modulars	80
§6. Strict convexity	82
§7. Uniform convexity	88
Appendix	92

Introduction

Let R be a universally continuous semi-ordered linear space¹⁾. A functional m on R is called a *modular*, if it satisfies the following modular conditions:

- (1) $0 \leq m(a) \leq \infty$ for all $a \in R$;
- (2) if $m(\xi a) = 0$ for all $\xi \geq 0$, then $a = 0$;
- (3) for any $a \in R$ there exists $\alpha > 0$ such that $m(\alpha a) < \infty$;
- (4) for every $a \in R$, $m(\xi a)$ is a *convex* function of ξ ;
- (5) $|a| \leq |b|$ implies $m(a) \leq m(b)$;
- (6) $a \frown b = 0$ implies $m(a+b) = m(a) + m(b)$;
- (7) $0 \leq a_\lambda \uparrow_{\lambda \in A} a$ implies $\sup_{\lambda \in A} m(a_\lambda) = m(a)$.

When a modular m is defined on R , R is called a *modulared semi-ordered linear space* with the modular m and is denoted by (R, m) , if necessary. We can define two kinds of norms on R by the formulas:

1) We use mainly notation and terminology of [12], [13].