ON CERTAIN PROPERTIES OF MODULAR CONVERGENCE

By

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Let R be a universally continuous semi-ordered linear space. A functional $m(a)$ $(a \in R)$ is said to be a modular on R if it satisfies the following modular conditions:

(1) $0 \leqq m(a) \leqq +\infty$ for all $a \in R$;

(2) if $m(\xi a)=0$ for all $\xi \geqq 0$, then $a=0$;

(3) for any $a \in R$ there exists $\alpha>0$ such that $m(\alpha a)<+\infty$;

(4) for every $a \in R$, $m(\xi a)$ is a convex function of ξ;

(5) $|a| \leqq |b|$ implies $m(a) \leqq m(b)$;

(6) $a \frown b=0$ implies $m(a+b)=m(a)+m(b)$;

(7) $0 \leqq a_\lambda \uparrow_{\lambda \in \Lambda} a$ implies $m(a)=\sup_{\lambda \in \Lambda} m(a_\lambda)$.

Throughout the paper we use the notations and terminologies used in [2]. Here $|w|\text{-}\lim_{\nu\to\infty} a_\nu = a$ or $w\text{-}\lim_{\nu\to\infty} a_\nu = a$ for $a, a_\nu \in R$ $(\nu=1,2,3,\cdots)$ means $\lim_{\nu\to\infty} |\bar{a}|(|a_\nu - a|)=0$ or $\lim_{\nu\to\infty} \bar{a}(a_\nu - a)=0$ respectively for any $\bar{a} \in \bar{R}^m$ [1].

If $\Phi(u)$ is a real convex function, defined for $u \geqq 0$, such that $\Phi(0)=0$ and $\Phi(u) \geqq 0$ for $u>0$, but $\Phi(u)$ not identically zero or infinity for $u>0$, then $\Phi(u)$ is called a YOUNG function.

We assume that Δ is a point set, and that a countably additive non-negative measure $\mu(E)$ $(E \in \Lambda)$ is defined for the σ-ring Λ of subsets of Δ. We suppose furthermore that the measure μ is complete (i.e. $\mu(E_1)=0$, $E_2 \subset E_1$ implies $E_2 \in \Lambda$, so $\mu(E_2)=0$), totally σ-finite (i.e. Δ is a countable union of sets of finite measure) and $\mu(\Delta)>0$.

If $f(x)$ is an arbitrary real-valued μ-measurable function on Δ, and $\Phi(u)$ is a YOUNG function, the space

$$L_\Phi = \left\{ f: \int_\Delta \Phi(\alpha |f(x)|) d\mu(x) < +\infty \quad \text{for some} \quad \alpha > 0 \right\}$$

is called ORLICZ space.

1) $\bar{R}^m$ be the modular conjugate space of R, i.e. $\bar{R}^m$ is the space of the modular bounded universally continuous linear functionals on R. The conjugate modular $\bar{m}$ of m is defined as $\bar{m}(\bar{a}) = \sup_{x \in R} \{\bar{a}(x) - m(x)\}$ for every $\bar{a} \in \bar{R}^m$.