SUBSPACE THEORY OF AN n-DIMENSIONAL SPACE WITH AN ALGEBRAIC METRIC

By

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Introduction. Let $F_n^{(p)}$ be an n-dimensional Finsler space with the metric given by the differential form of order p: $ds^p = a_{\alpha_1\cdots\alpha_p} dy^{\alpha_1}\cdots dy^{\alpha_p}$ (α's run over $1, 2, \cdots, n$), $a_{{}_1\alpha\cdots\alpha_p}$ being function of y's. Suppose that one has a homogeneous polynomial of order p in ξ's:

$$a = a_{\alpha_1\cdots\alpha_p} \xi^{\alpha_1}\cdots\xi^{\alpha_p} \tag{0.1}$$

which is defined in an n-dimensional projective space E_n attached to at a point (y). When we put $a_\alpha = \dfrac{1}{p}\dfrac{\partial a}{\partial \xi^\alpha}$ the resultant of the n forms a_α $(\alpha = 1, \cdots, n)$ is named the discriminant of the form a, and denoted by $\mathfrak{A}$. It is well known [1][1)] that $\mathfrak{A}$ is a homogeneous polynomial of order $n(p-1)^{n-1}$ in the coefficients $a_{\alpha_1\cdots\alpha_p}$, and that $\mathfrak{A}=0$ is the necessary and sufficient condition in order that n hypersurfaces $a_\alpha = 0$ in E_n have common point. Consequently, $\mathfrak{A}$ is a scalar density of weight $\omega = p(p-1)^{n-1}$. The differential geometry in $F_n^{(p)}$ was studied for $F_2^{(p)}$ by A. E. Liber [2] and for $F_3^{(3)}$ by the present author [3] and Yu. I. Ermakov [4]. Moreover, Yu. I. Ermakov [5] has established the foundation of differential geometry in general case: $F_n^{(p)}$ $(p>3)$ by introducing the affine connection $\Gamma^\alpha_{\beta\gamma}$. The principal purpose of the present paper is to discuss the theory of subspace immersed in $F_n^{(p)}$ $(p \geqq 3)$. §1 is devoted to the abridgment of the method of determination of the affine connection which was studied by Yu. I. Ermakov. §2 is offered to introduce the projection factor B^i_α and the normal vectors $\overset{p}{C}{}^\alpha$ and $\overset{p}{C}_\alpha$ to the subspace which will play the important roles in the theory of subspace. §3 and §4 are devoted to discuss the curvatures of a curve in the subspace and the Gauss and Codazzi equations for the subspace.

Furthermore we can discuss other many theories of the subspace making use of the projection factors and the normal vectors as well as the subspace in the Riemannian space. However we will omitt those discussions in this paper.

1) Numbers in brackets refer to the references at the end of the paper.