ON SEMI-LINEAR NORMAL BASIS

By

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§ 1. Introduction

As an extension of a normal basis theorem in Galois theory of fields T. Nakayama obtained the following so-called semi-linear normal basis theorem: Let L be a finite separable Galois extension field of a subfield K with the Galois group $\mathfrak{G}$, and let F be a subfield of L (not necessarily containing K) such that $F^{\mathfrak{G}} \subseteqq F$. Then there exists in L an element whose $[L:K]$ conjugates with respect to L are linearly independent over F or form a generating system of L over F according as $[L:F] \geqq$ or $\leqq [L:K]$. And he extended the theorem to the case of Galois extensions of division rings under some conditions [7, Assumptions I, II].

The main purpose of this paper is to extend the theorem to the case of strictly Galois extensions of division rings (Theorem 3).

On the other hand, F. Kasch considered in [5] the normal basis theorem in Noether's sense (Die zweite Fassung des Satz von der Normalbasis) for Galois extensions of division rings and obtained a necessary and sufficient condition under which the theorem holds [5, Satz 9]. His result will be easily generalized to the case of our semi-linear normal basis.

In §2, as a preliminary, we shall give a theorem on projective modules over a ring with minimum condition due to Nakayama and Nagao which serves as a main tool in our present considerations (Theorem 1). In §3, we shall give the proof of Theorem 3 and try to generalize some of Kasch's results. As an application of Theorem 1, we shall give in §4 a proof of a theorem on finite dimensional central division algebras due to Brauer and Albert.

§ 2. Preliminary results on projective modules over rings with minimum condition

Throughout this section, we assume that R is a ring with a unit element 1 and R satisfies the minimum condition for right ideals. As is well known R is decomposed into a direct sum of finite number of directly indecomposable right ideals e_iR, $i=1, \cdots, r$.