SOME THEOREMS ON GALOIS THEORY OF SIMPLE RINGS

By

Takasi NAGAHARA and Hisao TOMINAGA

Recently, in [7], we have succeeded in constructing Galois theory of simple rings under the assumption that the extension R/S considered is hereditarily Galois (h-Galois) and locally finite. However, we have believed that [7, Theorem 2.1] and [7, Theorem 3.1] should be stated under more desirable assumptions. One of the purposes of the present paper is to give a settlement to this problem. Concerning [7, Theorem 2.1], one will see that the assumption that R/S is locally finite can be excluded from those assumed there (Theorem 1). On the other hand, as was shown in [7, Lemma 3.4], if R/S is h-Galois and locally finite then $\mathfrak{G}R_r$ is dense in $\mathrm{Hom}_{S_l}(R, R)$. In §**1**, one will see also that if R/S is locally finite and $\mathfrak{G}R_r$ is dense in $\mathrm{Hom}_{S_l}(R, R)$ then the fundamental theorems in Galois theory of finite dimension hold still for regular intermediate rings of R/S left finite over S (Theorems 2 ond 3). And, if R/S is locally finite, $\mathfrak{G}R_r$ dense in $\mathrm{Hom}_{S_l}(R, R)$, and $V_R(V_R(S'))$ is simple for each regular intermediate ring S' of R/S with $[S':S]_l<\infty$, then [7, Theorem 3.1] is still valid even for a regular intermediate ring R' of R/S (Theorem 6). The proof of this improvement will be given in §**3**. §**2** is devoted exclusively to the treaty of algebraic Galois extensions, which is our second purpose. In fact, Theorem 4 may be regarded as a complete extension of [2, Theorem 3] to simple rings as well as an improvement of [7, Lemma 1.9]. §**2** contains also a sharpening of [7, Lemma 1.10] (Theorem 5).

Throughout the present paper, $R=\sum_1^n De_{ij}$ be a simple ring, where e_{ij}'s are matrix units and $D=V_R(\{e_{ij}\text{'s}\})$ is a division ring. And S be always a simple subring of R (containing 1 of R), $\mathfrak{G}$ the group of all the S-(ring) automorphisms of R. Further, we set $C=V_R(R)$, $Z=V_S(S)$, $V=V_R(S)$, $H=V_R(V)$ and $C_0=V_V(V)$. As to general notations and terminologies used here we follow the previous paper [7].

1. Now, we shall begin our study with the following lemma.

Lemma 1. *Let S be a regular subring of R, and $\mathfrak{H}$ a subgroup of $\mathfrak{G}$ containing $\tilde{V}$. If T is an intermediate ring of R/S left finite over S such that R is T-R-irreducible, and $T'=J(\mathfrak{H}(T, R))$, then* $\infty>[(\mathfrak{H}|T)R_r : R_r]_r$