AN INVERSE THEOREM OF GROSS'S STAR THEOREM

Dedicated to Prof. Kinjiro Kunugi on his 60th birthday

By

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Let $w=w(z)$ be an analytic function of z in a Riemann surface R whose values fall on the w-sphere. Let $z=z^{-1}(w)$ be its inverse. Let $e(w, w_0)$ be an arbitrary regular element of $z^{-1}(w)$. We continue analytically $e(w, w_0)$, using only regular element (without any algebraic element) along every ray: $\arg(w-w_0)=\theta$ $(0\leqq\theta<2\pi)$ toward infinity. Then, there arise two cases whether the continuation defines a singularity ω_θ in a finite distance or not, in the former case, we call the ray a singular ray. For each singular ray: $\arg(w-w_0)=\theta$, we exclude the segment between the singularity ω_θ and $w=\infty$ from the w-plane. The remaining domain Ω is clearly a (single valued) regular branch of $z=z^{-1}(w)$. Let $\rho=\rho(\theta)$ the polar coordinate of the singularity ω_θ or ∞ according as the singular ray exists or not. Then $\rho(\theta)$ is clearly lower semicontinuous and $S_n=E[\theta:\rho(\theta)\leqq n]$ is closed. We call the set $E[\theta:\rho(\theta)<\infty]$ the singular set S of Ω. Then by $S=\sum_{n=1}^{\infty} S_n$ S is an F_σ set. Then the famous Gross's Star Theorem is as follows:

Theorem. *Let R be a domain such that $R=E[z:|z|<\infty]$ in the z-plane and let $f(z)$ be an analytic function of $z\in R$ whose values fall on the w-plane. Let Ω be a star domain. Then S is a set of linear measure zero.*

This theorem was extended by M. Tsuji[1] to the case: R is a domain in the z-plane such that the boundary of R is a set of capacity zero and also extended by Z. Yûjôbo[2] to the case: R is a Riemann surface with null-boundary. The method used by them is essentially the same as used by W. Gross. On the other hand, T. Yoshida[3] showed that the Gross's theorem holds for not only conformal mappings but also for quasiconformal mappings

1) M. TSUJI: Theory of meromorphic functions in a neighbourhood of a closed set of capacity zero, Jap. Journ. Math., 19 (1944–1948).

2) Z. YUJÔBÔ: On the Riemann surfaces, no Green function of which exists, Math. Japonicae, 2 (1951).

3) T. YOSHIDA: On the behaviour of a pseudo-regular functions in a neighbourhood of a closed set of capacity zero, Proc. Japan Acad., 26 (1950).