ON QUASI-GALOIS EXTENSIONS OF DIVISION RINGS

Dedicated to Prof. Kinjiro Kunugi on his 60th birthday

By

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Throughout the present paper, R be always a division ring, and S a division subring of R. And, we use the following conventions: $C=V_R(R)$, $V=V_R(S)$, $H=V_R^2(S)=V_R(V_R(S))$, and further for any subrings $R_1\supseteq R_2$ of R the set of all R_2-(ring) isomorphisms of R_1 into R will be denoted as $\Gamma(R_1/R_2)$. As to other notations and terminologies used in this paper, we follow the previous one [3]. We consider here the following conditions:

(I) If S' is a subring of R properly containing S with $[S':S]_l<\infty$ then $\Gamma(S'/S)\neq 1$.

(I_0) If S' is a subring of R properly containing S with $[S':S]_r<\infty$ then $\Gamma(S'/S)\neq 1$.

(I′) H/S is Galois.

(II) If $S_1\supseteq S_2$ are intermediate rings of R/S with $[S_1:S]_l<\infty$ then $\Gamma(S_1/S)\,|\,S_2=\Gamma(S_2/S)$.

(II_0) If $S_1\supseteq S_2$ are intermediate rings of R/S with $[S_1:S]_r<\infty$ then $\Gamma(S_1/S)\,|\,S_2=\Gamma(S_2/S)$.

(II′) If $T_1\supseteq T_2$ are intermediate rings of R/H with $[T_1:H]_l<\infty$ then $\Gamma(T_1/S)\,|\,T_2=\Gamma(T_2/S)$[1].

(II_0') If $T_1\supseteq T_2$ are intermediate rings of R/H with $[T_1:H]_r<\infty$ then $\Gamma(T_1/S)\,|\,T_2=\Gamma(T_2/S)$.

Following [5], R/S is said to be *(left-)quasi-Galois* when (I) and (II) are fulfilled. Symmetrically, if (I_0) and (II_0) are done, we shall say R/S is *right-quasi-Galois*. In [5], we can find some fundamental theorems of quasi-Galois extensions. The purpose of the present paper is to expose several additional theorems concerning such extensions. At first, we shall recall the following lemmas which have been obtained in [4] and [5].

Lemma 1. *If S' is an intermediate ring of R/S then* $[V:V_R(S')]_r\leqslant$

1) In [5], the condition that if T is an intermediate ring of R/H with $[T:H]_l<\infty$ then $\Gamma(T/S)\,|\,H=\Gamma(H/S)$ was cited as (II′). However, it will be rather natural to alter it like above.