A REMARK ON MAZUR-ORLICZ'S NORM

By

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1. Let Ω be a measure space with finite measure μ, and let $M(u, v)$ be a real valued function which is defined on $[0, \infty)\times\Omega$ such that

(M. 1) $0\leqq M(u, v)\leqq +\infty$ for $(u, v)\in[0, \infty)\times\Omega$ with $M(0, v)=0$ a.e. $v\in\Omega$;

(M. 2) $M(u, v)$ is an increasing function and left-hand continuous with $\lim_{u\to 0} M(u, v)<+\infty$ a.e. $v\in\Omega$;

(M. 3) $M(u, v)$ is a measurable function of u for a fixed $v\in[0, \infty)$;

(M. 4) $\lim_{u\to\infty} M(u, v)>\lim_{u\to 0} M(u, v)$ a.e. $v\in\Omega$.

Now, we shall consider the function space $L_{M(u,v)}$ whose element f is as follows:

$$\rho(\alpha f)=\int_{\Omega} M(\alpha|f(v)|, v)d\mu <+\infty \quad \text{for some } \alpha>0. \tag{1}$$

If we identify f and g when $f(v)=g(v)$ except a measure zero set, then we can consider $L_{M(u,v)}$ as a conditionally complete vector lattice with a functional ρ[1]:

$$\rho(f)=\int_{\Omega} M(|f(v)|, v)d\mu. \tag{2}$$

In the case that $M(u, v)=M(u)$ for every $v\in\Omega$, and $\lim_{u\to 0} M(u)=0$, Mazur and Orlicz has considered in his paper [2], the quasi-norm such that

$$||f||=\inf\left\{\varepsilon;\ \rho\left(\frac{f}{\varepsilon}\right)<\varepsilon\right\}. \tag{3}$$

$||\cdot||$ has the following properties:

(F. 1) $||f+g||\leqq||f||+||g||$ for $f, g\in L_{M(u)}$;

(F. 2) $\alpha\to 0$, then $||\alpha f||\to 0$ for each $f\in L_{M(u)}$;

(F. 3) $||f||\to 0$, then $||\alpha f||\to 0$ for every real number α;

(F. 4) $0\leqq f\leqq g$, then $||f||\leqq||g||$;

(F. 5) $0\leqq f_1\leqq f_2\leqq\cdots$, $\sup_n ||f_n||<+\infty$, then $\bigcup_{n=1}^{\infty} f_n\in L_{M(u)}$ and $||\bigcup_{n=1}^{\infty} f_n||=\sup_n ||f_n||$;

1) This space is an example of quasi-modular spaces. cf. [3].