NOTE ON DECOMPOSITION SETS OF SEMI-PRIME RINGS

By

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Introduction. As has been observed by Jacobson the set $\mathfrak{P}=\mathfrak{P}(A)$ of all primitive ideals of a ring A may be made into a topological space endowed with Stone's topology, and recently, concerning topological properties of the structure space, Suliński [8] obtained some structure theorems of a semi-simple ring which is represented as a subdirect sum of simple rings with unity.

In this note, we shall extend his results to semi-prime rings and give necessary and sufficient conditions for a semi-prime ring to have a minimal decomposition set.

§ 1. First of all, we shall prove the following extension of [1, Theorem 1].

Lemma 1. *Let T be an ideal of a ring A.*

(1) *If p is a prime ideal of A then $T\frown p$ is a prime ideal of the ring T and if moreover p does not contain T then $(p\frown T\colon T)$*[1] *$=p$.*

(2)[2] *If p_1 is a prime ideal of the ring T, then there exists a prime ideal p of A such that $p\frown T=p_1$ and, if $p_1\neq T$, then $(p_1\colon T)=p$.*

Proof. (1) By [6, Lemma 2], $T\frown p$ is a prime ideal of the ring T. Assume that p does not contain T. Then $T\cdot(p\frown T\colon T)\subseteqq p$ implies $(p\frown T\colon T)\subseteqq p$ and hence we have $(p\frown T\colon T)=p$.

(2) Let B be the ideal of A generated by p_1 and let x be an arbitrary element of $B\frown T$. Since $xTxTx\subseteqq TBT\subseteqq p_1$ and p_1 is a prime ideal in T, x belongs to p_1, and hence $T\frown B=p_1$. The complement C of p_1 in T is an *m-system* (in T whence) in A and does not meet B. By Zorn's lemma, there exists a prime ideal p of A containing B such that p does not meet C and satisfies $T\frown p=p_1$. Moreover, if $p_1\neq T$ then p can not contain T, and hence, by (1), we have $(p_1\colon T)=p$.

A ring A is called *a semi-prime ring* if it is isomorphic to a subdirect sum of prime rings, i.e., if there exist prime ideals p_α $(\alpha\in\Lambda)$ of

1) We shall denote by $(p\frown T\colon T)$ the set $\{a\in A;\ Ta\subseteqq p\frown T\}$.

2) Cf. [3] and [7].