ON THE EXISTENCE OF FUNCTIONS OF EVANS'S TYPE

by

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We proved the following

Theorem[1]. *Let R be a Riemann surface with null-boundary. Then there exists a harmonic function $U(z)$ such that $U(z)$ has a negative logarithmic pole at $p\in R$ and $U(z)\to\infty$ as z tends to the ideal boundary.*

Recently M. Nakai extended the above theorem as follows:

Theorem[2]. *Let R be a Riemann surface with positive boundary. Let $G(z,p)$ be the Green's function of R with pole at p. Put $G_\delta=E[z\in R: G(z,p)>\delta]$. Then there exists a harmonic function in R such that $D(\min(M, U(z)))\leqq M\alpha<\infty$ and $U(z)\to\infty$ as z tends of the boundary of R in G_δ for any $\delta>0$ and that any positive harmonic function $V(z)(\leqq U(z))$ must be zero, where α is a conotant.*

We consider the existence of functions of Evans's type for more general sets and obtain some results which contain the above two theorems as their special applications.

1. Generalized Green's function[3]. Let R be a Riemann surface with positive boundary. Let R_n $(n=1, 2, \cdots)$ be its exhaustion with compact relative boundary ∂R_n. Let G[4] be a subsurface in R. Let $w_{n,n+i}(z)$ be a harmonic function in $R_{n+i}-(G\cap(R-R_n))$ such that $w_{n,n+i}(z)=0$ on $\partial R_{n+i}-G$ and $w_{n,n+i}(z)=1$ on $G\cap(R-R_n)$. We call $\lim_n \lim_i w_{n,n+i}(z)$ the harmonic measure (H.M.) of the boundary $(B\cap G)$ determined by G and denote it by $w(G\cap B, z, R)$. As for a set F in R. Let $w(F, z, R)$ be the least positive harmonic function in $R-F$ and $=1$ on F. We call $w(F, z, R)$ H.M. (harmonic measure) of F. Let $G_1\supset G_2$ be subdomains in R. Let $\omega_{n,n+i}(z)$ be a harmonic function

1) Z. KURAMOCHI: Mass distributions on the ideal boundaries of abstract Riemann surfaces, I. Osaka Math. J., 8, 119-138 (1956).

2) M. NAKAI: Green potential of Evans type on Royden's compactification of a Riemann surface. Nagoya Math. J., 24, 205-239 (1964).

3) Z. KURAMOCHI: On harmonic functions representable by Poisson's integral. Osaka Math. J., 10, 103-117 (1958).

4) In this paper relative boundary ∂G of a subsurface G consists of enumberably infinite number of analytic curves clustering nowhere in R.