A CHARACTERIZATION OF STRONGLY SEPARABLE ALGEBRAS*)

By

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§ 1. In his paper [2] T. Kanzaki introduced the notion of strongly separable algebras over a commutative ring and obtained an interesting characterization of such algebras [2, Theorem 1].

Throughout the present note, $A \ni 1$ will represent always an algebra over a commutative ring $R \ni 1$, and C the center of A.

Let P be the set of elements $\sum x_i \otimes y_i$ in $A \underset{R}{\otimes} A$ such that $\sum x_i x \otimes y_i = \sum x_i \otimes x y_i$ for all x in A, and let φ be the A–A-(module-) homomorphism of $A \underset{R}{\otimes} A$ into A defined by $\varphi(\sum x_i \otimes y_i) = \sum x_i y_i$. If $\varphi(P)$ contains 1, then A is said to be a strongly separable (R-) algebra. (The definition is somewhat different from the original one in [2]. But, as is easily seen, the two definitions are equivalent).

The purpose of the present note is to give an another characterization of a strongly separable algebra A when it is R-finitely generated and projective (Theorem 2).

§ 2. A is said to be a Frobenius (resp. symmetric) algebra if A is a finitely generated, projective R-module and there exists an A-isomorphism: $A_A \cong \mathrm{Hom}_R(A, R)_A$ (resp. ${}_A A_A \cong {}_A\mathrm{Hom}_R(A, R)_A$), where $\mathrm{Hom}_R(A, R)$ is regarded as an A–A-module by the following operations:

$$bfa(x) = f(axb) \qquad a, b, x \in A, \ f \in \mathrm{Hom}_R(A, R)$$

At first we shall quote here the following theorem which is due to Kanzaki [2, Theorem 1].

Theorem 1. *A is a strongly separable (R-) algebra if and only if A is a separable (R-) algebra and $A = C \oplus [A, A]$, as C-module, where $[A, A]$ is the C-submodule of A generated by all $xy - yx$ (x, y in A).*

Lemma 1. *Let $B \ni 1$ be a ring, and Z the center of B. If $B = Z \oplus [B, B]$ as Z-module, then $\mathrm{Hom}_Z(B, Z)^B$ (the set of elements f in $\mathrm{Hom}_Z(B, Z)$ such that $af = fa$ for all a in B) is a free Z-module of rank 1.*

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