FINITE OUTER GALOIS THEORY OF NON-COMMUTATIVE RINGS

By

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§ 0. Introduction. It is the purpose of this paper to extend the Galois theory of commutative rings given by S. U. Chase, D. K. Harrison and A. Rosenberg [4] to non-commutative case. In what follows, for the sake of simplicity, we shall state main results for directly indecomposable rings: Let $A \ni 1$ be a directly indecomposable ring, G a finite group of automorphisms of A, and $B=A^G=\{x\in A\,;\ \sigma(x)=x$ for all σ in $G.\}$. We call A/B a G-Galois extension if there are elements $a_1, \cdots, a_n\,;\ a_1^*, \cdots, a_n^*$ in A such that $\sum_i a_i\cdot\sigma(a_i^*)=\delta_{1,\sigma}(\sigma\in G)$, where $\delta_{1,\sigma}$ means Kronecker's delta. If $V_A(B)=C$ (the center of A), then A/B is a G-Galois extension if and only if the mapping $x\otimes y\to xy$ from $A\otimes_B A$ to A splits as an A-A-homomorphism (Th. 1.5). Let A/B be a G-Galois extension, and A' a G-invariant subring of A, i.e., $\sigma(A')=A'$ for all σ in G, and put $B'=A'^G$. If A'/B' is a G-Galois extension and $B'_{B'}$ is a direct summand of $A'_{B'}$, then there hold the following. (1) For any subgroup H of G, $A^H=B\otimes_{B'}A'^H=A'^H\otimes_{B'}B$. (2) Let $\{\bar{T}\}$ be the set of all G-invariant intermediate rings of A/A', and $\{T\}$ the set of all intermediate rings of B/B' such that $A'T=TA'$. Then, $\bar{T}\to\bar{T}\cap B$ and $T\to A'T=TA'$ are mutually converse order isomorphisms between $\{\bar{T}\}$ and $\{T\}$, and $\bar{T}/(\bar{T}\cap B)$ is a G-Galois extension (Th. 5.1).

Let A/B be a G-Galois extension, $V_A(B)=C$, and B_B a direct summand of A_B. Then there hold the following: (1) G coincides with the set of all B-automorphisms of A (Th. 4.2). (2) For any subgroup H of G, $\{\sigma\in G\,;\ \sigma|A^H=1_{A^H}\}=H$. (3) If T is an intermediate ring of A/B, the following are