

ON THE TOPOLOGY OF SOME ALMOST CONTACT MANIFOLDS

Dedicated to Professor A. Komatsu
for his 60th birthday

By

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1. Introduction and results.

We mean by an *almost contact structure* of a real orientable differentiable manifold M^{2n+1} a reduction of the structural group $SO(2n+1)$ of the tangent bundle $\tau(M)$ to $U(n) \times 1$. (Cf. J. W. Gray [2] and S. Sasaki [6].) If M has the almost contact structure, we say also that M is *almost contact*. Gray has proved that a 5-dimensional real orientable manifold is almost contact if and only if its third integral Stiefel-Whitney class is zero. The almost contact structure is quite analogous to an almost complex structure. Passing to stable class of the tangent bundle both determine a stable complex structure. Recently, the study of stable complex structures of real vector bundles to find almost complex structures of even dimensional orientable differentiable manifolds is advanced by E. Thomas [10] and W. A. Sutherland [8] and others. In this note, we deduce some results on the existence of almost contact structures using their results. Our main theorem is,

Theorem 1.1. *Let M be a real orientable differentiable manifold of odd dimension. The tangent bundle $\tau(M)$ has a stable complex structure if and only if M is almost contact.*

This theorem is proved by considering an induced map of Postnikov systems for fibre maps between classifying spaces. Our technique follows D. W. Kahn [4]. Let w_i denote the i -th Stiefel-Whitney class and δ the Bockstein coboundary operator associated with the exact sequence $Z \rightarrow Z \rightarrow Z_2 \rightarrow 0$. Let a be a point of M . If $M - a$ has an almost contact structure then we say that M has an *almost contact structure except a point*. As applications of Theorem 1.1, we obtain the following:

Theorem 1.2. *Let M denote a closed real orientable differentiable manifold of dimension 9 such that $w_4(M) = 0$. Then M has an almost contact structure except a point if and only if*