Hokkaido Mathematical Journal Vol. 12 (1983) p. 461-471

Particle path length estimates for the Navier Stokes equations in three space dimensions

By G. F. D. DUFF

(Received January 14, 1983)

Abstract. Flows with finite energy of a viscous incompressible fiuid in a domain of three dimensional space are studied to estimate particle path lengths. In the general case a bound is given for the essential maximum path length as time $T\rightarrow\infty$. If the domain satisfies a Poincaré inequality, then as $T\rightarrow\infty$ all particle motions are essentially uniformly bounded. Some additional asymptotic results are also given.

1. Introduction

In this paper are given results on the path lengths of the motions of fluid elements or point masses, which we may refer to as particles.

For three space dimensions solutions of the Navier Stokes equations may be turbulent in the sense of Leray [7] and all calculations must be made with allowance for this possibility which remains not completely decided despite many advances in the mathematical theory of nonlinear fluid motions. Any turbulent solutions may develop singularities which however can only occur on sets of low dimension in space and time [2, 8]. One must therefore work with integrals that remain convergent in the presence of such possible singularities. After a sufficiently long time interval, the singularities can no longer appear, and emphasis is then on asymptotic behaviour. We show that motions generated by initial values with finite energy give rise to finite path lengths over bounded time intervals. An asymptotic estimate is given for large elapsed times in the general case of a domain in R^3 that satisfies a cone condition. When the domain also satisfies a Poincaré inequality, it is shown that the path lengths are bounded as $T\rightarrow\infty$. These results extend and complete those announced in [3] and are in turn based on a technique used by Foias, Guillopé and Temam in [4]. For domains with boundary we use orthogonal projection on the subspace of solenoidal vector functions that vanish on the boundary, and this is essentially our other necessary condition on the domains considered. The method yields an estimate free of boundary terms as stated in Theorem 1 below. This is applied to the path