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On the global existence and asymptotic behavior of solutions of reaction-diffusion equations

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1. Introduction

Let Ω be a bounded domain with smooth boundary Γ in R^{n}. Let $\beta\geq 1$, and $\mu_{j}>0$ $(j=1,2)$. R. Martin posed a problem on the existence and uniform bounds of solutions $u=\{u_{1}, u_{2}\}$ of the reaction-diffusion equation of the form:

$$(1)\qquad \begin{cases} \dfrac{\partial u_{1}}{\partial t}=\mu_{1}\Delta u_{1}-u_{1}u_{2}^{\beta} \\ \dfrac{\partial u_{2}}{\partial t}=\mu_{2}\Delta u_{2}+u_{1}u_{2}^{\beta} \end{cases} \qquad x\in\Omega,\ t>0$$

under various boundary conditions and non-negative initial data; this equation is related to the Rosenzweig-MacArthur equation in ecology (see J. Maynard-Smith [5]; D. Conway and A. Smoller [2]). N. Alikakos [1] obtained L^{∞}-bounds of solutions of (1) subject to the homogeneous Neumann boundary condition under the assumption $1\leq\beta<(n+2)/n$, and gave a positive partial answer to it. The purpose of the present paper is to give a complete answer to the problem of Martin.

We consider a solution $u=\{u_{1}, u_{2}\}$ of the more general type of reaction-diffusion equations

$$(2)\qquad \frac{\partial u_{j}}{\partial t}=\mu_{j}\Delta u_{j}+f_{j}(u),\ x\in\Omega,\ t>0\ (j=1,2);$$

subject to the boundary condition:

$$(3)\qquad \alpha_{j}(x)\frac{\partial u_{j}}{\partial n}+\left(1-\alpha_{j}(x)\right)u_{j}=0,\ x\in\Gamma,\ (j=1,2);$$

and with the initial condition:

$$(4)\qquad u_{j}|_{t=0}=a_{j}(x),\qquad x\in\Omega\ (j=1,2),$$

($\partial/\partial n$ denotes differentiation in the direction of the exterior normal to Γ). Here we make the following assumptions:

ASSUMPTION 1. *$\alpha_{j}(x)$ $(j=1,2)$ is a non-negative C^{2}-function on Γ such*