Hokkaido Mathematical Journal Vol. 12 (1983) p. 343-359

On the distribution of the poles of the scattering matrix for two strictly convex obstacles

Mitsuru IKAWA

(Received December 1, 1982)

§ 1. Introduction

Let $\mathcal{O}$ be a bounded open set in $\boldsymbol{R}^3$ with sufficiently smooth boundary Γ. We set $\Omega=\boldsymbol{R}^3-\bar{\mathcal{O}}$. Suppose that Ω is connected. Denote by $\mathcal{S}(z)$ the scattering matrix for an acoustic problem

$$(1.1)\qquad \begin{cases} \dfrac{\partial^2 u}{\partial t^2}-\Delta u=0 & \text{in} \quad \Omega\times(-\infty,\infty) \\ u(x,t)=0 & \text{on} \quad \Gamma\times(-\infty,\infty) \end{cases}$$

where $\Delta=\sum_{j=1}^{3}\dfrac{\partial^2}{\partial x_j^2}$. Concerning the definition and the fundamental properties of $\mathcal{S}(z)$, see Lax and Phillips [6, Chapter V]. It is well known that it is holomorphic in $\{z;\ \mathrm{Im}\, z\leqslant 0\}$ and meromorphic in the whole complex plane $\boldsymbol{C}$ as $\mathcal{L}(L^2(S^2), L^2(S^2))$[1] valued function (Theorem 5.1 of Chapter V, [6]), and the problem to clarify the relationship between the geometric propoerties of the obstacles and the location of the poles of the scattering matrix is important and has been interested, but we know only a few works about the existence of the poles [1, 3, 8].

Assuming that

$$(1.2)\qquad \begin{cases} \mathcal{O}=\mathcal{O}_1\cup\mathcal{O}_2, \qquad \bar{\mathcal{O}}_1\cap\bar{\mathcal{O}}_2=\phi, \\ \text{and the Gaussian curvature of } \Gamma_j \text{ the boundary of } \mathcal{O}_j, \\ j=1,2 \text{ vanishes nowhere.} \end{cases}$$

Bardos, Guillot and Ralston [1] proved the existence of infinitely many poles in a region $\{z;\ \mathrm{Im}\, z\leqslant\varepsilon\log(|z|+1)\}$ for any $\varepsilon>0$.[2] They introduced the notion of pseudo-poles $\alpha_{m,\vec{m}}$ and used it to prove the above result, but it was not considered the problem that the pseudo-poles do approximate the actual poles of $\mathcal{S}(z)$. In [3, 4] we gave a precise information on the location

1) $\mathcal{L}(E,F)$ denotes the set of all linear bounded operator from E to F.

2) Though the condition (1.2) is assumed in [1], they used only a certain condition on the Poincaré mapping at the periodic broken ray. Petkov [8] gives a genelaization of [1].