Positively curved complex submanifolds immersed in a complex projective space II

Dedicated to Professor Y. Katsurada on her 60th birthday

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1. Introduction

Let $P_m(\boldsymbol{C})$ be a complex projective space of complex dimension m with the Fubini-Study metric of constant holomorphic sectional curvature 1. Recently S. Tanno [6] has proved the following result.

PROPOSITION A. *Let M be an n-dimensional complete complex submanifold immersed in $P_{n+p}(\boldsymbol{C})$. If every holomorphic sectional curvature of M with respect to the induced metric is greater than $1-\frac{n+2}{6n^2}$, then M is complex analytically isometric to a linear subspace $P_n(\boldsymbol{C})$.*

In this paper we shall prove the following theorems.

THEOREM 1. *Let M be an n-dimensional complete complex submanifold immersed in $P_{n+p}(\boldsymbol{C})$. If every Ricci curvature of M with respect to the induced metric is greater than $n/2$, then M is complex analytically isometric to a linear subspace $P_n(\boldsymbol{C})$.*

Theorem 1 is the best possible in this direction.

THEOREM 2. *Let M be an n-dimensional complete submanifold immersed in $P_{n+p}(\boldsymbol{C})$. If every holomorphic sectional curvature of M with respect to the induced metric is greater than δ, then M is complex analytically isometric to a linear subspace $P_n(\boldsymbol{C})$, where*

$$\delta=\begin{cases}\dfrac{3n-1}{3n+1} & (n\leq 5)\\[2ex] \dfrac{2n-3}{2n-2} & (n>5).\end{cases}$$

Theorem 2 is an improvement of Proposition A.

THEOREM 3. *Let M be an n-dimensional complete complex submanifold immersed in $P_{n+p}(\boldsymbol{C})$. If $n\geq 2$ and if every sectional curvature of M with respect to the induced metric is greater than δ, then M is complex analytically*

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