On a pseudo umbilical submanifold in a Riemannian manifold of constant curvature

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Introduction.

H. Liebmann [8] has proved that an ovaloid with constant mean curvature in a 3-dimensional Euclidean space is a sphere. The above problem for a closed hypersurface in a Riemannian manifold has been generalized by Y. Katsurada [3], [4] and K. Yano [17]. Y. Katsurada [5], [6], H. Kôjyô [5], T. Nagai [6], [12] and K. Yano [18] have given conditions for a submanifold of codimension greater than 1 in a Riemannian manifold to be pseudo umbilical by making use of integral formulas.

On the other hand M. Okumura [13] has proved that a submanifold of codimension 2 in an odd dimensional sphere is totally umbilical under certain coditions. To prove the above result, M. Okumura made use of the fact that the structure tensor of the natural normal contact structure on the odd dimensional sphere is a conformal Killing tensor of order 2 which has been defined by S. Tachibana [15].

In the previous papers [9], [10], the present author proved for a submanifold of codimension p in a sphere and a Riemannian manifold of constant curvature respectively that the submanifold is totally umbilical under certain conditions by making use of integral formulas. However, in the papers, it has been assumed that the connection of the normal bundle is trivial.

In this paper, the present author studies on a submanifold of codimension p in a Riemannian manifold of constant curvature without the condition that the connection of the normal bundle is trivial and proves that the submanifold is pseudo umbilical.

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§1. Conformal Killing tensors.

Recently S. Tachibana [15] and T. Kashiwada [2] have introduced the notion of conformal Killing tensor field in a Riemannian manifold. They discussed such the tensor and obtained some results.

Let $\widetilde{M}^{n+p}$ be a $(n+p)$-dimensional Riemannian manifold with the metric