Some topological properties of certain Riemannian manifolds with positive curvature

By Toshio SAKAGUCHI

Introduction.

Let M be a compact and simply connected Riemannian manifold with positive curvature K, $0<K\leqq 1$. Denote by $d(p, q)$ the distance between two points p and q of M. K. Hatsuse has introduced the following number $L(M)$;

$$L(M)=\operatorname*{Max}_{p,q,r\in M}\left\{d(p, q)+d(q, r)+d(r, p)\right\}.$$

Actually it is possible to define $L(M)$ for any compact and connected Riemannian manifold with positive curvature. It has been studied by K. Hatsuse and Y. Tsukamoto [4],* [5] to investigate the topological structure of M satisfying suitable conditions for $L(M)$. In particular, K. Hatsuse has proved the following theorem.

THEOREM. *Let M be a compact and simply connected Riemannian manifold with positive curvature K, $0<K\leqq 1$. If $L(M)<3\pi$, then M is homeomorphic to a sphere. In particular, if $L(M)=2\pi$, then M is isometric to the sphere with constant curvature* 1.

The purpose of the present paper is to prove the following theorems.

THEOREM A. *Let M be a compact and connected Riemannian manifold with positive curvature K, $0<K\leqq 1$. If $L(M)=2\pi$ and there exist two points p and q of M satisfying $d(p, q)=\pi$, then M is isometric to the sphere with constant curvature* 1.

THEOREM B. *Let M be an n-dimensional ($n\geqq 2$) compact and connected Riemannian manifold which is not simply connected. Suppose that the sectional curvature K of M satisfies the inequalities $1/4<\delta\leqq K\leqq 1$, where δ is a constant, and the fundamental group $\pi_1(M)$ of M satisfies $\pi_1(M)=Z_2$. If $L(M)=3\pi/2$, then M is isometric to the real projective space $PR^n(1)$ of constant curvature* 1, *and if $L(M)=3\pi/2\sqrt{\delta}$, then M is isometric to $PR^n(\delta)$ of constant curvature δ.*

§1 will be of reviews of definitions and notations and §2 will be devoted

* Numbers in brackets refer to the references at the end of the paper.