Discrimination of the space-time V. I.

By Hyôitirô TAKENO

§ 1. Introduction.

The present paper is a continuation of [1][1], [2] and [3], and deals with the problem of the discrimination of the space-time V. In other words, we are going to establish a theory by which we can determine whether a four-dimensional Riemannian space-time defined by g_{ij} arbitrarily given in any coordinate system is a V or not. Mathematically speaking, this problem is "to determine the necessary and sufficient condition that the given g_{ij} be reducible to the form

$$ds^2 = -dx^2 - Bdy^2 - Cdz^2 + Ddt^2, \tag{1.1}$$

where B, C and D are positive valued functions of x alone".

As is easily understood, the problem of "determining whether a given space-time is V or not" is not only interesting from the standpoint of tensor analysis but also its solution is of importance when we consider the physical meanings of the given space-time. If the answer of this problem is given by some tensor equations to be satisfied by the curvature tensor K_{ijmn} made from g_{ij}, especially when the equations contain no tensor other than g_{ij}, η_{ijmn} $(=\sqrt{-g}\,\epsilon_{ijmn})$ and K_{ijmn}, we may say that the problem is solved in the most desirable form. Unfortunately, however, we have not succeeded in finding such equations at the present stage of the investigations. In the present paper and the forthcoming one [4], we shall give another way of discriminating V using the theory of characteristic system (abbreviated to c.s.) developed in [1], [2] and [3].

If we see the results of [1], it is true that if we can determine whether or not there exists a c.s. satisfying (F_1), (F_2) and (F_3) below, the purpose of the discrimination may be attained. But in order to carry out this plan, we need some devices and techniques. Now let g_{ij} be an arbitrary fundamental tensor whose signature is of type $(- - - +)$, and U be the space-time defined by this g_{ij}. Determine from g_{ij} the forms to be taken by the characteristic vectors (abbreviated to c.v.) assuming that the U is a V. If only these forms are known, we can easily determine whether the U is a V or not by substituting them into the fundamental equations (F_1), (F_2) and

1) Numbers in brackets refer to the references at the end of the paper.