Integral formulas for closed submanifolds in a Riemannian manifold

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Introduction.

In the previous paper [9][1] we have given certain generalization of integral formulas of Minkowski type and obtained some properties of a closed orientable hypersurface in a Riemannian manifold. For a submanifold in a Riemannian manifold Y. Katsurada, T. Nagai and H. Kôjyô [7], [8] obtained the following

THEOREM A (*Y. Katsurada and T. Nagai*) *Let R^n be a Riemannian manifold which admits a vector field ξ^i generating a continuous one-parameter group G of homothetic transformations in R^n and V^m a closed orientable submanifold in R^n such that*

(i) *its first mean curvature $H_1 = const.$,*

(ii) *the inner product $\underset{E}{n}_i\xi^i$ has fixed sign on V^m,*

(iii) *the generating vector ξ^i is contained in the vector space spanned by m independent tangent vectors and Euler-Schouten unit vector $\underset{E}{n}^i$ at each point on V^m,*

(iv) $R_{ijhk}\underset{E}{n}^i\underset{E}{n}^h g^{\alpha\beta}B^j_\alpha B^k_\beta \geqq 0$ *at each point on V^m.*

Then every point of V^m is umbilic with respect to the vector $\underset{E}{n}^i$.[2]

THEOREM B (*Y. Katsurada and H. Kôjyô*) *Let R^n be a space of constant curvature which admits a vector field ξ^i generating a continuous one-parameter group G of conformal transformations in R^n and V^m a closed orientable submanifold in R^n such that*

(i) *its first mean curvature $H_1 = const.$,*

(ii) *the inner product $\underset{E}{n}^i\xi^i$ has fixed sign on V^m,*

(iii) *the generating vector ξ^i is contained in the vector space spanned by m indepent tangent vectors and $\underset{E}{n}^i$ at each point on V^m.*

Then every point of V^m is umbilic with respect to the vector $\underset{E}{n}^i$.

THEOREM C (Y. K*atsurada and* H. K*ôjyô*) *Let R^n be a space of con-*

1) Numbers in brackets refer to the references at the end of the paper.

2) With respect to R_{ijhk}, $\underset{E}{n}^i$, $g^{\alpha\beta}$and B^i_α refer to §1 of the present paper.