A note on homotopy spheres

By Yoshifumi ANDO

§ 0. Introduction

All manifolds will mean compact oriented smooth manifolds without any further notices. In [5] J. Milnor and M. Kervaire has determined bP_m, the group of homotopy spheres which bound parallelizable manifolds. If $m=4k$ $(k\neq 1)$, then bP_{4k} is the cyclic group of order $\sigma_k/8$. In this paper we will consider the group of homotopy spheres which bound manifolds of dim m, whose Spivk normal fiber spaces are trivial. We denote it by bF_m. We show that there exists an analogy of the above fact for bF_m. We define bF_m^0 to be the group of homotopy spheres which bound manifolds of dim m whose Spivak normal fiber spaces are trivial and whose indexes are zero. Then bF_m^0 is a subgroup of bF_m. Let f_k be 1/8 min $\{n\in Z \mid n$ is the index of a closed manifold of dim $4k$ whose Spivak normal fiber space is trivial. $n>0\}$. Then we have

THEOREM 0. 1 i) *If $m\geq 6$, then $bF_m=bP_m$.* ii) *If $m=4k$, then the group bF_m/bF_m^0 is isomorphic to a cyclic group of order f_k.*

THEOREM 0. 2 *Let d_{2n} be the greatest common divisor of* $2^{4n-2}(2^{4n-1}-1)$ numerator $(B_m/4m)$ *and* $2\left\{2^{2n-1}\cdot(2^{2n-1}-1)\cdot a_n\cdot \text{numerator}\left(\frac{B_n}{4n}\right)\right\}^2$. *Then $f_{2n}\leq d_{2n}$. Especially if $k=1$, then $f_2=4$ which is equivalent to $bF_8^0\cong Z_7$.*

§ 1 is devoted to preliminaries. Theorem 0. 1 will be proved in § 2. In § 3 we will give some computations and a proof of Theorem 0. 2.

§ 1. Preliminaries

We quote some results due to D. Sullivan [8]. Let $F/0$ be the fiber of the map, $BSO\rightarrow BSF$. Let W be a simply connected manifold with a boundary $\partial W\neq\phi$. Let $hS(W)$ denote the concordance classes of h-smoothings h: $(W', \partial W')\rightarrow(W, \partial W)$ of W.

(1. 1) If dim $W\geqq 6$, then there is a bijection η, $hS(W)\rightarrow[W, F/0]$. Moreover if a h-smoothing, h: $(W', \partial W')\rightarrow(W, \partial W)$ corresponds to f: $W\rightarrow F/0$ by η, then the stable tangent bundle $\tau_{w'}$ of W' is equivalent to $h^*\tau_w\oplus h^*f^*(\gamma)$, where γ is a universal $F/0$-bundle [8, 9].

If ∂W is a homotopy sphere of dim $\partial W=m-1$, let a map $\bar{d}$; $hS(W)\rightarrow\theta_{m-1}$ be defined as follows. θ_{m-1} denotes the group of homotopy spheres of