A conformal transformation and a special concircular scalar field in a Riemannian manifold

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Introduction

Recently T. Koyanagi [1][2)] has investigated some properties of a Riemannian manifold which admits a scalar field ρ characterized by the property

$$\nabla_l \rho_k = \sigma \rho g_{kl}, \qquad \sigma = \textit{non-zero constant}, \tag{0. 1}$$

(such a scalar field ρ is called the special concircular scalar field) where $\rho_k = \nabla_k \rho$ and g_{kl} means the metric tensor of the manifold. He obtained the following

THEOREM A. *Let M be a Riemannian manifold of dimension n which has the curvature tensor satisfying*

$$\nabla_{[m}\nabla_{l]} R_{hijk} = 0$$

and admits the special concircular scalar field ρ *defined by* (0. 1). *Then M is of constant curvature.*

COROLLARY A_1. *Let M be an n-dimensional Einstein space* $(n>2)$ *which has the scalar curvature* $R \neq 0$, *the curvature tensor such that*

$$\nabla_{[m}\nabla_{l]} R_{hijk} = 0$$

and admits a proper conformal Killing vector field ξ^i. *Then M is of constant curvature.*

THEOREM B. *Let M be a Riemannian manifold of dimension n which has the Ricci tensor such that*

$$\nabla_{[l}\nabla_{h]} R_{jk} = 0$$

and admits the special concircular scalar field ρ. *Then M is an Einstein space.*

A conformal Killing vector field ξ^i satisfies an equation;

$$\underset{\xi}{\mathcal{L}} g_{ij} = \nabla_j \xi_i + \nabla_i \xi_j = 2\rho g_{ij},$$

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2) Numbers in brackets refer to the references at the end of the paper.