# A Characterization of Conway's Group $C_3$

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## § 1. Introduction

In this paper we characterize the Conway's simple group $C_3$ of order $2^{10}\cdot 3^7\cdot 5^3\cdot 7\cdot 11\cdot 23$ by the structure of the centralizer of a noncentral involution.

Main theorem. *Let $G$ be a finite group satisfying the following properties*:

( *i* ) *$G$ has an involution $e$ with $C_G(e)\cong Z_2\times M_{12}$,*

( *ii* ) *$e\in O^2(G)$.*

*Then $G\cong C_3$.*

The centralizer of a central involution of the Conway's group $C_3$ is isomorphic to the perfect central extention of $S_p(6,2)$ by a group of order 2. The main difficulty in proving the main theorem is in the determination of the structure of a $S_2$-subgroup of $G$. If this is established, we can easily know that $G$ has the same involution fusion pattern and the centralizer of a central involution as the Conway's group $C_3$. Thus the characterization theorem of $C_3$ by *D*. Fendel [1] implies that $G\cong C_3$.

Throughout, all group considered are finite. Most of our notations are standard (see [2]) and we use the "bar" convention for homomorphic images. Furthermore we use the following notations:

| | |
|---|---|
| $x\sim y$ | $x$ is conjugate to $y$, |
| $a:\ x\longrightarrow y$ | $y=x^a=a^{-1}xa$, |
| $x^H$ | $=\{x^h \mid h\in H\}$, |
| $\langle x^H\cap K\rangle$ | $=\langle y \mid y\in K,\ x\sim y \text{ in } H\rangle$, |
| $A*B$ | the central product, |
| $A \wr B$ | the wreathed product. |

## § 2. Preliminalies

**A. Mathieu group $\mathbf{M_{12}}$.** We list some properties of Mathieu group $M_{12}=M$. Let $c$ be an involution of the center of a $S_2$-subgroup of $M$.

(1) *Generators and relations of the centralizer of $c$.*

$C_M(c)=\langle a_1, a_2, b_1, b_2, s, t\rangle$,

$a_1^2=a_2^2=b_1^2=b_2^2=[a_1, a_2]=[b_1, b_2]=\mathrm{c}$,