On the holomorphically projective correspondence between Kählerian spaces preserving complex structure

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Introduction.

Let (M^n, g) and $(M'^n, g')(n>2)$ be n-dimensional Riemannian spaces with positive definite metric tensors g and g', respectively. The celebrated Beltrami's theorem (see [1][1] §40) of classical differential geometry states that if (M^n, g) is of constant curvature then the space which is in projective correspondence with it is necessarily of constant curvature. N. S. Sinyukov [5] has generalized the above theorem to the non-affine projective correspondence of (M^n, g) with (M'^n, g') which is locally symmetric. Further W. Roter [4] and J. Robinson and J. D. Zund [3] have generalized N. S. Sinyukov's theorem to the case when (M'^n, g') is recurrent.

On the other hand T. Ōtsuki and Y. Tashiro [2] have studied holomorphically projective correspondences between Kählerian spaces. Let (M^n, g, F) and (M'^n, g', F') be Kählerian spaces of real dimension $n(=2m>2)$ with the structures (g, F) and (g', F'), respectively. Then they have proved the generalization of Beltrami's theorem, that is, if (M^n, g, F) is of constant holomorphic sectional curvature then the space which is in holomorphically projective correspondence with it preserving complex structure is necessarily of constant holomorphic sectional curvature.

The purpose of the present paper is to investigate what restrictions the requirement that (M^n, g, F) and (M'^n, g', F') be in holomorphically projective correspondence preserving complex structure imposes on these spaces. An example of such a restriction is given by the above result of T. Ōtsuki and Y. Tashiro.

The author wishes to express here his sincere thanks to Professor Yoshie Katsurada and Doctor Tamao Nagai for their kindly guidances and advices.

§1. Preliminaries.

We shall first give preliminary formulas on the Kählerian space and the holomorphically projective correspondence between Kählerian spaces.

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1) Numbers in brackets refer to the references at the end of the paper.