A note on minimal submanifolds in Riemannian manifolds

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In this note we shall prove the following: Let $\bar{M}^{n+p}$ be a Riemannian manifold of constant curvature $\bar{c}$, and let M^n be a minimal submanifold in $\bar{M}$ of constant curvature c. Then either M is totally geodesic, i.e. $\bar{c}=c$, or $\bar{c}\geqq(2p-n+1)c/(p-n+1)$, in the latter case the equality arising only when $\bar{c}>0$. Our method is based on the Simons' type formula which has been given by Simons [4].

On the other hand, we shall study the Laplacian of the Ricci operator of a minimal submanifold of codimension 1 in a Riemannian manifold of constant curvature and give some inequality. And combing the theorems of Lawson [2], we shall prove some theorems for compact minimal hypersurfaces in a unit sphere.

1. Preliminaries

In this section we shall summarize the basic formulas for submanifolds in Riemannian manifolds.

Let $\bar{M}$ be a Riemannian manifold of dimension $n+p$, and let M be a submanifold of $\bar{M}$ of dimension n. Let $\langle\,,\rangle$ be the metric tensor field of $\bar{M}$ as well as the metric induced on M. We denote by $\bar{\nabla}$ the covariant differentiation in $\bar{M}$ and by ∇ the covariant differentiation in M determined by the induced metric on M. Then the Gauss-Weingarten formulas are given by

$$\bar{\nabla}_X Y=\nabla_X Y+B(X, Y), \qquad X, Y\in\mathfrak{X}(M),$$
$$\bar{\nabla}_X N=-A^N(X)+D_X N, \quad X\in\mathfrak{X}(M), \quad N\in\mathfrak{X}(M)^{\perp}$$

Where D is the linear connection in the normal bundle $T(M)^{\perp}$. We call A and B the second fundamental form of M and they satisfy $<B(X, Y), N>=<A^N(X), Y>$. The Riemannian curvature tensors of $\bar{M}$ and M will be denoted by $\bar{R}$ and R respectively. From the Gauss-Weingarten formulas, we have

$$\bar{R}_{X,Y}Z=R_{X,Y}Z-A^{B(Y,Z)}(X)+A^{B(X,Z)}(Y)+(\tilde{\nabla}_X B)(Y, Z)-(\tilde{\nabla}_Y B)(X, Z),$$

where $\tilde{\nabla}$ denotes the covariant differentiation for B. And we obtain the