# Operational calculus for tensor products of linear operators in Banach spaces*,†

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## Introduction

Given two matrices $A$ and $B$ and a polynomial $P(\xi,\eta)=\sum_{j,k} c_{jk}\xi^j\cdot\eta^k$ ($c_{jk}$ complex numbers) in $\xi$ and $\eta$, a polynomial operator $P(A\otimes I, I\otimes B)=\sum_{j,k} c_{jk}A^j\otimes B^k$ is defined, where $I$ is the identity matrix. Stéphanos proved (see [13]) that the set of the eigenvalues of $P(A\otimes I, I\otimes B)$ coincides with the set of the complex numbers $\sum_{j,k} c_{jk}\alpha^j\beta^k$ with $\alpha$ eigenvalue of $A$ and $\beta$ eigenvalue of $B$.

The aim of the present paper is to extend this result and to prove the spectral mapping theorem for tensor products of densely defined closed linear operators in complex Banach spaces. We develop an operational calculus defined for holomorphic functions of several variables in a neighbourhood of the product of the extended spectra of those operators, which may be considered roughly as the tensor product of the operational calculi defined for holomorphic functions of one variable developed by I. Gelfand, Dunford and Taylor ([15], [3], [20], [8]). Brown and Pearcy [2] have proved for bounded operators $A$ and $B$ on a Hilbert space and $P(\xi,\eta)=\xi\cdot\eta$ that the spectrum of the tensor product $A\otimes B$ is the set of the products of the spectra of $A$ and $B$. Ichinose [9] has extended it for some unbounded operators $A$ and $B$ in Banach spaces. The result of Stéphanos for several bounded operators in Banach spaces has also been obtained by Schechter [19]. Our results include those of Brown and Pearcy, Schechter and partly Ichinose. The spectral mapping theorem enables us, in particular, to find the resolvent of a polynomial operator defined for tensor products of closed operators, from the knowledge of the spectra of those operators.

Section 1 deals with the basic notions about linear operators, their maximal extensions and their spectra, and tensor products of spaces and operators.

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