Iterated mixed problems for d'Alembertians II

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§1. Introduction and results

The aim of this note is to establish a generalization of the results in the previous paper [2]. Definitions and terminologies in [2] will be used here also.

Let $(t, x)=(t, x', x_n)$ be variables in the $(n+1)$-dimensional Euclidean space $\boldsymbol{R}^{n+1}$ and (τ, σ, λ) $(\tau=\xi-i\gamma)$ the dual variables of (t, x', x_n) respectively. Furthermore let χ be a permutation of m letters $1, \cdots, m$. In the open half space $\boldsymbol{R}_+^{n+1}=\{x_n>0\}$ with boundary $\boldsymbol{R}^n=\{x_n=0\}$, we then consider an iterated mixed problem $(P, {}^{\chi}B_j)$ for d'Alembertians:

$$P(t, x; D_t, D_x)u=f \qquad \text{in } \boldsymbol{R}_+^{n+1},$$
$${}^{\chi}B_j(t, x'; D_t, D_x)u=g_j \qquad (j=1, \cdots, m) \qquad \text{on } \boldsymbol{R}^n.$$

Here we shall recall the definitions of P and ${}^{\chi}B_j$ ([2], §1):

$$P^0(t, x; \tau, \sigma, \lambda)=\prod_{j=1}^{m} P_j^0(t, x; \tau, \sigma, \lambda),$$
$$P_j^0(t, x; \tau, \sigma, \lambda)=-\tau^2+a_j(t, x)^2(\lambda^2+|\sigma|^2),$$
$$0<a_m(t, x)<\cdots<a_1(t, x),$$
$$\chi=\begin{pmatrix}1, 2, \cdots\cdots, m\\ k_1, k_2, \cdots, k_m\end{pmatrix},$$
$${}^{\chi}B_1^0(t, x'; \tau, \sigma, \lambda)=B_{k_1}^0(t, x'; \tau, \sigma, \lambda),$$
$${}^{\chi}B_j^0(t, x'; \tau, \sigma, \lambda)$$
$$=(B_{k_j}^0)(t, x'; \tau, \sigma, \lambda)\prod_{h=1}^{j-1} P_{k_h}^0(t, x', 0; \tau, \sigma, \lambda) \qquad (j\geqq 2),$$
$$B_j^0(t, x'; \tau, \sigma, \lambda)=\lambda-\sum_{k=1}^{n-1} b_{jk}(t, x')\sigma_k-c_j(t, x')\tau,$$

where b_{jk} and c_j are real valued and Q^0 denotes the principal part of a differential operator Q. We are concerned with L^2-well posed problems ([2], §2) and hence the solution of our problem has zero initial data on $t=0$ provided that $f=0$ and $g_j=0$ in $t<0$.

In order to state the main results we recall a classification of L^2-well posed problems of second order with constant coefficients ([2], §1). We say a problem to be type U if it satisfies uniform Lopatinski condition and