On a generalization of F. M. Markel' theorem

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The purpose of this paper is to prove the following result.

THEOREM. *Let G be a non-identity finite group satisfyin the following conditions* (*a*) *and* (*b*): (*a*) *If the orders of centrlizers of two elements are equal, they are conjugate in G.* (*b*) *A Sylow 2-subgroup of G is abelian. Then G is isomorphic to the symmetric group of degree* 3.

The notation in this paper is standard. (See. D. Gorenstein [2])

LEMMA 1. *Let G be a finite group satisfying the condition* (*a*). Then x is cojugate to x^k, for any element x in G, where k is prime to the order of x.

PROOF. Obvious.

LEMMA 2. *Let G be a finite solvable group, S a Sylow 2-subgroup of G. If the order of x is odd and x is in $N_G(S)$, then x is a non-real element.*

PROOF. We prove by induction on the order of G. Let K be a minimal normal subgroup of G. If K does not contain x, the image of x in G/K is non-real by induction. So x is non-real in this case. If K contains x, we have that $[x, S] \subseteq S \cap K = 1$. Since $C_G(x)$ contains a Sylow 2-subgroup of G, we have that the order of $N_G(<x>)/C_G(x)$ is odd. Hence x is not a real element in this case, too.

Now we separate the proof of the theorem into two parts; G is solvable and G is nonsolvable.

Part (1). G is solvable.

LEMMA 3. $G = O_{2',2}(G)$.

PROOF. By lemma 2, we have that $N_G(S) = S$. Since Sylow 2-subgroups of G are abelian, Burnside' transfer theorem implies the lemma.

LEMMA 4. *A Sylow 2-subgroup of G is elementary abelian.*

PROOF. Obviaus.

From now on we use the following notation throughout this paper; $H = O(G)$. We fix a prime p such that $O^p(H) \lneqq H$ and set $H_0 = O^p(H)$.

LEMMA 5. $p = 3$.

PROOF. Let U be the inverse image of the Frattini subgroup of H/H_0