The endomorphism ring of an indecomposable module with an Artinian projective cover

By Tsutomu TAKEUCHI

As is well known, each endomorphism of an Artinian, Noetherian and indecomposable module is either nilpotent or an automorphism. C. I. Vinsonhaler has proved that each endomorphism of an indecomposable module with a Noetherian injective hull is either nilpotent or an automorphism (see [6], [7] and [5; p. 75]). In this note we shall verify that each endomorphism of an indecomposable module with an Artinian projective cover is either nilpotent or an automorphism.

We will assume throughout that M denotes a nonzero unital left R-module, where R is a nonzero ring with identity.

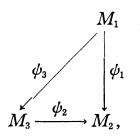
THEOREM. Let M be an indecomposable left R-module and let P be an Artinian left R-module which is a projective cover of M with a minimal epimorphism $\varphi: P \rightarrow M$. Then each endomorphism α of M is either nilpotent or an automorphism.

REMARK 1. M is called a semiperfect module iff every factor module of M has a projective cover (cf. Mares [4]).

Let N be a submodule of M. A cocomplement N^c of N in M is a minimal submodule of M such that $N+N^c=M$. M is called a cocomplemented module iff every submodule of M has a cocomplement in M.

As is easily proven, semiperfect modules are cocomplemented. Conversely, projective cocomplemented modules are semiperfect (see Kasch-Mares [3]).

REMARK 2. In the following commutative diagram with left *R*-modules M_i and homomorphisms ϕ_i (i=1, 2, 3):



if ψ_1 is an epimorphism and if ψ_2 is a minimal epimorphism, then ψ_3 is necessarily an epimorphism.