On Jacobi fields in quaternion Kaehler manifolds with constant Q-sectional curvature

By Mariko KONISHI

Kosmanek [6] gave a characterization of Kaehler manifolds of constant holomorphic sectional curvature in relation with Jacobi fields. That is, the following property ($\mathscr{C}\mathscr{J}$) is satisfied if and only if the Kaehler manifold is of constant holomorphic sectional curvature:

($\mathscr{C}\mathscr{J}$) "*For a given geodesic $\gamma(t)$ in a Kaehler manifold (J, g), every Jacobi field Y along γ such that $Y(0)=0$ and $\nabla_{\dot\gamma}Y(0)=J\dot\gamma(0)$, is proportional to $J\dot\gamma$, where $\dot\gamma(t)$ denotes the tangent vector at $\gamma(t)$*".

The main purpose of this paper is to study the corresponding problem in quaternion Kaehler manifolds and characterize the manifolds of constant Q-sectional curvature, that is to prove Theorem 1.

On the other hand, Kashiwada [4] recently obtained analogous result for Sasakian manifolds (ϕ, ξ, g) with constant ϕ-holomorphic sectional curvature in terms of Jacobi field along geodesics orthogonal to ξ. From a point of view of submersion [8], the results for Kaehler manifolds and Sasakian manifolds are closely related and so are the relations between quaternion Kaehler manifolds and manifolds with Sasakian 3-structure $(\{\xi, \eta, \zeta\}, \tilde g)$. We apply Theorem 1 to study Jacobi fields in the manifolds with Sasakian 3-structure when each ϕ-, ψ- and θ-holomorphic sectional curvatures are constant on the distribution $\tilde D=\{\tilde X \mid \tilde g(\xi, \tilde X)=\tilde g(\eta, \tilde X)=\tilde g(\zeta, \tilde X)=0\}$.

§ 1. Quaternion Kaehlerian manifolds

Let M be a differentiable manifold of dimension n and assume that there is a 3-dimensional vector bundle V consisting of tensors of type (1.1) over M satisfying the condition:

"In any coordinate neighborhood U of M, there is a local base $\{F, G, H\}$ of V such that

$$(1.1)\qquad \begin{aligned} &F^2=G^2=H^2=-I\,, \\ &GH=-HG=F\,, \qquad HF=-FH=G\,, \qquad FG=-GF=H\,, \end{aligned}$$

I denoting the identity tensor field of type (1.1) in M".

In an almost quaternion manifold (M, V), we take two intersecting