Homotopy groups of s. s. complexes of embeddings

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§1. Introduction

Recently codimension 3 embeddings have been discussed in many papers. It is known that codimension 3 topological embeddings are approximated by PL embeddings and close (topological or PL) embeddings are isotopic (e.g. [2]).

In this paper we shall compare homotopy groups of s. s. complexes of topological embeddings with those of PL embeddings, and show that the local contractibility of spaces of topological embedding implies similar properties of PL embeddings with codimension $\geqq 3$.

Main results are followings.

THEOREM A. *Suppose that* M^m, Q^q *are* m-dim. *and* q-dim. *PL manifolds, and* $\mathcal{E}^{TOP}(M, Q)$ (*resp.* $\mathcal{E}^{PL}(M, Q)$) *is a Kan complex of locally flat topological* (*resp. PL*) *embeddings. Then*

1) *If* $q-m \leqq 2$ $q \geqq 5$, $\mathcal{E}^{PL}(M, Q)$ *is homotopy equivalent to* $\mathcal{E}^{TOP}(M, Q)$ *provided* $H^i(M, Z_2)=0$ *for* $i \leqq 3$.

2) *If* $q-m \geqq 3$ *or* $q \leqq 3$, $\mathcal{E}^{PL}(M, Q)$ *is homotopy equivlent to* $\mathcal{E}^{TOP}(M, Q)$.

THEOREM B. *When* $q-m \geqq 3$, $\mathcal{E}^{PL}(M, Q)$ *is locally* n*-connected for any* $n \geqq 0$.

This gives a partial answer to the question of Edward ([2]). The case when $n \leqq q-m-3$ is obtained by Lusk ([8]).

By rB^k and rS^{k-1} we denote the k-ball with radius $r\{(X_1, \cdots, X_k) \in R^k \mid |X_i| \leqq r\}$ and the $(k-1)$-sphere $\partial(rB^k)$, respectively. R^k is identified with the subspace $R^k \times 0 \subset R^{k+1}$.

ASSUMPTIONS. In this paper we assume the followings: A map denoted by

$$f\colon M \times \Delta^k \longrightarrow Q \times \Delta^k$$

is level preserving i.e. $pr_2 \circ f = pr_2$, f_t is given by $f(x, t) = (f_t(x), t)$, and $f(M \times \Delta^k) \subset \operatorname{int} Q \times \Delta^k$. It is valid to generalize the result in this paper for proper embeddings. Embeddings and immersions are to be locally flat. For manifolds M^m and Q^q, we assume $m < q$ because $m = q$ case is obtained