Metrics induced by capacities and boundary behaviors of quasiconformal mappings on open Riemann surfaces

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Introduction.

M. Nakai (cf. [4]) proved that every quasiconformal mapping between two open Riemann surfaces can be homeomorphically extended to their Royden compactifications. It is well-known (cf. [2]) that the Royden compactification is not metrizable. In this paper we shall study the homeomorphic extensibility of quasiconformal mappings between two open Riemann surfaces to their metrizable compactifications. To do so we shall introduce a new metric $d=d_R$ on an open Riemann surface R induced by the Kuramochi capacity on R. Our main results are the followings:

(i) Let R be an open Riemann surface. If each Kuramochi kernel $\tilde{g}_b$ with pole b on the Kuramochi boundary of R is unbounded, then the completion of R with respect to d is compact.

(ii) Let R_1 and R_2 be two open Riemann surfaces. If both R_1 and R_2 satisfy the assumption in (i), then every quasiconformal mapping from R_1 onto R_2 can be homeomorphically extended over their completions with respect to d.

1. Metrics induced by capacities.

Let R be an open Riemann surface. We say that a closed curve in R joining $a\in R$ and $b\in R$ means a continuous mapping γ: $z=z(t)$ of $[0, 1]$ into R such that $z(0)=a$ and $z(1)=b$. We write $\gamma=\{z(t);\ 0\leqq t\leqq 1\}$ for simplicity. We denote by $\Gamma_{a,b}=\Gamma_{a,b}(R)$ the family of all closed curves in R joining a and b.

A non-negative finite real-valued function Φ on the family of all compact subsets on R is said to be a capacity in the sense of G. Choquet if it satisfies the following properties:

(a) If $K_1\subset K_2$, then $\Phi(K_1)\leqq\Phi(K_2)$.

(b) $\Phi(K_1\cup K_2)+\Phi(K_1\cap K_2)\leqq\Phi(K_1)+\Phi(K_2)$.

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