Q-connections and their changes on an almost quaternion manifold

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Affine connections of certain types on an almost quaternion manifold were studied by M. Obata ([9], [10]), and the existence of affine connections such that the almost quaternion structure is covariantly constant with respect to their connections, the transformations preserving the almost quaternion structure, and so forth were discussed. Recently, S. Ishihara ([6]) has defined the quaternion Kählerian manifold by using the tensor calculus, and interesting results have been obtained by several authors ([1], [2], [3], [4], [6], [7], [8], [12]).

In the present paper, we shall define Q-connections satisfying the condition which the Riemannian connection on the quaternion Kählerian manifold is imposed on, and show the existence of Q-connections and the change of Q-connections preserving the Q-projective curvature tensor field which is analogous to the H-projective change ([5], [11], [14]).

Throughout this paper, we assume that manifolds, fields and connections are differentiable and of class C^∞, the indices $a, b, c, \cdots, j, k, l$ run over the range $\{1, \cdots, n\}$ and the summation convension will be used.

§ 1. Q-connections.

Let M be a manifold of dimension $n\,(=4m)$, and assume that there is a 3-dimensional vector bundle V consisting of tensors of type (1, 1) over M satisfying the following condition:

In any coordinate neighborhood of M, there is a local base $\{F, G, H\}$ of V such that

$$(1.1)\qquad \begin{cases} F^2 = G^2 = H^2 = -I, \\ FG = -GF = H, \qquad GH = -HG = F, \qquad HF = -FH = G, \end{cases}$$

where we denote by I the identity tensor field of type (1, 1) on M. Then, the bundle V is called an almost quaternion structure on M, and (M, V) an almost quaternion manifold. From now on, we shall discuss in the local and use this local base $\{F, G, H\}$ of V, whose local components are