A generalization of Whitney Lemma

By Kazuaki KOBAYASHI

(Received June 5, 1975)

§0. In this paper we study the elimination of the intersections of manifolds which is a generalization of WHITNEY LEMMA as follows.

WHITNEY LEMMA (*simply connected version*) (*see* [*R* & *S*]): *Let* P^p, Q^q *be a pair of connected compact locally flat submanifolds of* M^m *which are transverse, so that* $p+q=m$. *Suppose* (1) $p\geqq 3$, $q\geqq 3$ *and* $\pi_1(M)=0$ *or* (2) $p\geqq 2$, $q\geqq 3$ *and* $\pi_1(M-Q)=0$. *If the intersection number of* P *and* Q, $\varepsilon(P, Q)$, *is zero, we can ambient isotope* P *off* Q, *by an isotopy which has compact support.*

We work in the *PL* category ([Z]) throughout the paper.

MAIN RESULT I (BOUNDED VERSION) (COROLLARY TO THEOREM 1). *Let* P *be a compact* p*-manifold and* M *be an* m*-manifold. Let* Q *be a compact* q-dim. *submanifold of* M *and* $f: P\to \operatorname{Int} M$ *be an embedding, so that* $p+q=m+k$. *If* (1) $\partial P\neq\phi$, P *is* k*-connected,* $k\leqq p-3$ *and* $f(P)\cap Q\subset f(\operatorname{Int} P)$ *or* (2) $\partial Q\neq\phi$, Q *is* k*-connected,* $k\leqq q-3$ *and* $f(P)\cap Q\subset \operatorname{Int} Q$, *then there is an embedding* $g: P\to \operatorname{Int} M$ *which is ambient isotopic to* f *and* $g(P)\cap Q=\phi$.

MAIN RESULT II (CLOSED VERSION) (THEOREM 2.) *Let* P, M *be a connected closed* p*- and* m*-manifolds and* Q *be a connected closed* q*-submanifold of* M. *Let* $f: P\to M$ *be an embedding and let* $p+q=m+k$. *Put* $N=f(P)\cap Q$.

(1) *If* P, Q *are* k*-connected and* M *is* $(k+1)$*-connected and if* $k+3\leqq p$, $k+3\leqq q$ *then* P*-side and* Q*-side intersection classes* $\varepsilon_P(N)$ *and* $\varepsilon_Q(N)$ *are defined* (§2 *for definition*).

(2) *Suppose* $p, q\leqq m-3$ *and* $\varepsilon_P(N)=0$ *or* $\varepsilon_Q(N)=0$ *provided* $\min(p, q)\geqq 2k+3$ *or* $\varepsilon_i(N)=0$ *provided* $\max(p, q)\geqq 2k+3$ *where* $i=Q$ *if* $\max(p, q)=p$ *and* $i=P$ *if* $\max(p, q)=q$. *Then there is an embedding* $g: P\to M$ *so that* g *is ambient isotopic to* f *and* $g(P)\cap Q=\phi$.

(3) *If* P, Q *are* $(k+1)$*-connected and* M *is* $(k+2)$*-connected and if* $k+4\leqq p$, $k+4\leqq q$, $\varepsilon_P(N)$ *and* $\varepsilon_Q(N)$ *are uniquely determined for* N (*i.e. they do not depend on the choice of* K, L *and* J *at the definition of* $\varepsilon_P(N)$, $\varepsilon_Q(N)$).