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Remarks on L^2-well posedness of mixed problems for hyperbolic systems

By Kôji KUBOTA

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§ 1. Introduction and main theorem

In the present paper we are concerned with the boundary value problem for a $2m\times 2m$ strictly x_0-hyperbolic system P of order one:

$$(P, B)\begin{cases} P(x, D)u=f & \text{in } \Omega,\\ B(x')u=g & \text{on } \Gamma,\end{cases}$$

where Ω is the open half space $\{x=(x', x_n)=(x_0, x'', x_n)\,;\ x_0\in R^1, x''\in R^{n-1}, x_n>0\}$ $(n\geqq 2)$, its boundary Γ is noncharacteristic for P and $B(x')$ is an $m\times 2m$ matrix of rank m for every $x'\in\Gamma$. All coefficients of differential operators considered here are assumed to be smooth in $\bar{\Omega}$ and constant outside of a compact subset of $\bar{\Omega}$. Then the problem (P, B) is said to be *L^2-well posed* if and only if there exist positive constants γ_0 and C such that for every $\gamma\geqq\gamma_0$, $f\in H_{1,\gamma}(\Omega)$ and $g\in H_{3/2,\gamma}(\Gamma)$ (P, B) has a unique solution u in $H_{1,\gamma}(\Omega)$ satisfying

$$\gamma^2\|u\|_{0,\gamma}^2\leqq C\left(\|f\|_{0,\gamma}^2+|g|_{1/2,\gamma}^2\right). \tag{1.1}$$

This definition of L^2-well posedness is weaker than that in Kreiss [7], but it implies a certain well posedness of the corresponding mixed problem with initial data on $x_0=0$. (See § 5 in Kubota [8]).

In a recent paper Sato and Shirota [15] have refined the results at § 7 in Ohkubo and Shirota [11] who investigated the above problem under the condition that all of constant coefficients problems frozen the coefficients at Γ are L^2-well posed. In this paper we try to complete some of the results at §§ 5, 6 and 8 in [11]. Here we shall use the same terminologies as in [11] unless otherwise indicated, but we denote by $\sqrt{\zeta}$ the branch of square roots of ζ such that $\sqrt{1}=1$ (see [15]).

Throughout the present article we assume the following conditions (i) and (ii):

CONDITION (i) (with respect to the principal part P^0 of P). *Let* (η, σ, λ) *be the covector of* (x^0, x'', x_n). *Then for every* $(x, \eta, \sigma)\in\Gamma\times(R^n\backslash 0)$ *the*