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On conjugation families

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1. Introduction.

In his paper [2], Goldschmidt has proved a generalization of Alperin's theorem in [1]. The purpose of this paper is to give another proof of his result in [2], namely, to show that his family defined in [2] is a conjugation family.

Let p be a prime, G be a finite group, $Syl_p(G)$ denote the set of Sylow p-subgroups of G, and P be an element of $Syl_p(G)$. Let $\mathscr{S}$ be the set of all pairs (H, T) such that H is a nontrivial subgroup in P and T is a subgroup in $N_G(H)$.

Our notation corresponds to that of Alperin [1], Goldschmidt [2], and Glauberman [3]. Let $\mathscr{F}$ be a subset of $\mathscr{S}$, H be a subgroup of P, and L be a finite group.

a) Suppose that A and B are nonempty subsets of P and $g\in G$. We say that A is $\mathscr{F}$-conjugate to B via g if there exist elements $(H_1, T_1), \cdots, (H_n, T_n)$ in $\mathscr{F}$ and $g_1, \cdots, g_n$ in G such that $g_i\in T_i\,(i=1, \cdots, n)$, $A^g=B$, where $g=g_1\cdots g_n$, and $A\subseteq H$ and $A^{g_1\cdots g_i}\subseteq H_{i+1}\,(i=1, \cdots, n-1)$.

b) We say that $\mathscr{F}$ is a conjugation family (for P in G) if it has the following property: whenever A and B are nonempty subsets of P and $g\in G$ and $A^g=B$, then A is $\mathscr{F}$-conjugate to B via g.

c) We say that H is a tame intersection (in P) if $H=P\cap Q$ for some $Q\in Syl_p(G)$ and $N_P(H)\in Syl_p(N_G(H))$. In particular, there is a Sylow p-subgroup R of G such that $N_R(H)\in Syl_p(N_G(H))$ and $P\cap R=H$.

d) We say that L is p-isolated if, for some $S\in Syl_p(L)$, $\langle N_L(E)\colon 1\neq E\leq S\rangle$ is a nontrivial proper subgroup of L. In particular, if L is p-isolated, then there exists S_1 in $Syl_p(L)$ such that $S\cap S_1=1$.

THEOREM A.

For each $(H, N_G(H))\in\mathscr{S}$, *we assign a normal subgroup* K_H *of* $N_G(H)$. *Let* $\mathscr{F}$ *be the set of all pairs* $(H, T)\in\mathscr{S}$ *satisfying the following conditions* i), ii), iii), *and* iv).

i) *H is a tame intersection in P.*

ii) *$H=P$ or the factor group $N_G(H)/H$ is p-isolated.*