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A counter example of Gross' star theorem

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1. Introduction.

Let R be an open Riemann surface. Let $w=f(p)$ be a meromorphic function on the sufrace R. We denote by Φ_f the covering surface generated by the inverse function of $w=f(p)$ over the extended w-plane. Let $q_0 \in \Phi_f$ be a regular point on Φ_f lying over the basic point $w_0=f(p_0) \neq \infty$ and let l_θ be the longest segment on Φ_f that starts from q_0, consists of only regular points of Φ_f and lies over the half straight line $\arg(w-w_0)=\theta$ $(0 \leq \theta < 2\pi)$ on the w-plane. Here a regular point of Φ_f is a point of Φ_f not being an algebraic branch point. If l_θ has finite length, then l_θ is said to be a singular segment with its argument θ of Φ_f. The set $\Omega=\bigcup_{0\leq\theta<2\pi} l_\theta$ is clearly a domain and is called a Gross' star region with the center q_0 on Φ_f. We call the set $S_\Omega=\{e^{i\theta} \mid l_\theta \text{ is singular}\}$ the singular set of Ω. Since the set $S_n=\{e^{i\theta} \mid \text{the length of } l_\theta \leq n\}$ is a closed set on the unit circle $\Gamma=\{|w|=1\}$ and $S_\Omega=\bigcup_{n=1}^{\infty} S_n$, S_Ω is an F_σ set on Γ.

If for any Gross' star region Ω on Φ_f the linear measure $(dm=d\theta)$ of S_Ω equals zero, then we say that the function $f(p)$ or Φ_f has the Gross' property. Further, if any meromorphic function on R has the Gross' property, then we say that the Riemann surface R has the Gross' property. W. Gross [1] proved that $R=\{|z|<\infty\}$ has the Gross' property. And M. Tsuji [cf. 5] extended this Gross' theorem in the following: If R is a domain on the z-plane and the boundary of R is of logarithmic capacity zero, then R has the Gross' property. And Z. Yûjôbô [cf. 5] proved a Riemann surface $R \in O_g$ has the Gross' property. And Z. Kuramochi [2], [3] proved that there exists a Riemann surface $R \in O_{HP}$ such that R has not the Gross' property and also there exists a domain $R \in O_{AB}-O_g$ on the z-plane such that R has not the Gross' property.

Further Z. Kuramochi considered the next K. Noshiro's problem: *Is the singular set of any Gross' star region of a covering surface belonging to O_g a set of capacity zero?* Let H be an F_σ set on Γ. If there exists a sequence of closed sets $F_n (n=1, 2, \cdots)$ such that $H=\bigcup_{n=1}^{\infty} F_n$ and $F_n \cap \overline{H-F_n}=\phi$ for every n, then the F_σ set H is called a *discrete* F_σ set. Z. Kuramochi