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Remarks on relatively flat modules

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Introduction

Let R be a ring and I be an idempotent ideal of R. Put $\mathscr{T}=\{{}_RM; IM=M\}$, $\mathscr{F}=\{{}_RM; IM=0\}$, and $\mathscr{C}=\{{}_RM; \mathrm{Ann}_M I=0\}$. Then $(\mathscr{T}, \mathscr{F})$, $(\mathscr{F}, \mathscr{C})$ is a TTF-theory over the category of all left R-modules. Since I is an ideal, we can define a TTF-theory over the category of all right R-modules in the same way. We denote it $(\mathscr{T}', \mathscr{F}')$, $(\mathscr{F}', \mathscr{C}')$.

Bland calls a left R-module M relatively flat if, for any exact sequence $0\rightarrow A\rightarrow B\rightarrow C\rightarrow 0$ of right R-modules such that $C\in\mathscr{F}'$, a sequence $0\rightarrow A\otimes M\rightarrow B\otimes M\rightarrow C\otimes M\rightarrow 0$ is exact [1]. In this paper we call such a module I-flat.

It is well known that a flat module is characterized by the purity in the sense of Cohn. We shall define the I-purity and give the similar characterization of the I-flatness.

In section 2, we investigate the I-flatness of R/I-modules. We give the characterization of a ring which has the property that each I-flat module is codivisible with respect to $(\mathscr{T}, \mathscr{F})$.

Throughout this paper, all rings are associative with unit and all modules are unital.

The reader is referred to [5] about the torsion theories.

1. On I-purity

It is well known that the purity in the sense of Cohn and the flatness are closely related. We shall show that the similar relation holds between the I-purity and the I-flatness.

DEFINITION 1–1. *We call an exact sequence* $0\rightarrow L\rightarrow X\rightarrow M\rightarrow 0$ *of left R-modules I-pure if, for each* $A\in\mathscr{F}'$, *a sequence* $0\rightarrow A\otimes L\rightarrow A\otimes X\rightarrow A\otimes M\rightarrow 0$ *is exact.*

We call a submodule U of V I-pure if the induced sequence $0\rightarrow U\rightarrow V\rightarrow V/U\rightarrow 0$ *is I-pure.*

THEOREM 1–2. *The following conditions are equivalent for a left R-module M.*

(1) *M is I-flat.*