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On maximal hyperbolic sets

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In this paper we study structures of hyperbolic sets which are maximal invariant sets in their sufficiently small neighbourhoods. One of their important examples is an Anosov diffeomorphism (not necessarily assumed to be topologically transitive).

First we give some definitions.

DEFINITION. Suppose U is an open set of a manifold M, $f: U\rightarrow M$ is a diffeomorphism onto an open set of M. $\Lambda\subset U$ is called a hyperbolic set if Λ is a compact invariant set which satisfies the followings; $T_{\Lambda}M$ splits into a Whitney sum of Tf-invariant subbundles

$$T_{\Lambda}M=E^{s}\oplus E^{u}$$

such that there are $c>o$ and $o\leqq\lambda<1$ with

$$\|Tf^{n}v\|\leqq c\lambda^{n}\|v\| \qquad \text{if } v\in E^{s}$$

$$\|Tf^{-n}v\|\leqq c\lambda^{n}\|v\| \qquad \text{if } v\in E^{u}$$

for $n\geqq 0$.

Definition. Let $\mathscr{A}=\{A_{1}, A_{2}, \cdots A_{n}\}$ be a finite set, and $\mathscr{A}^{Z}$ be a space of functions from Z to $\mathscr{A}$ with the compact-open topology. (Here we assume $\mathscr{A}$ and Z have the discrete topologies.) Let

$$\rho:\mathscr{A}^{Z}\rightarrow \mathscr{A}^{Z}$$

be given by $\rho((a_{i})_{i\in Z})=(b_{i})_{i\in Z}$ where $b_{i}=a_{i+1}$. Let $T=(t_{ij})_{i,j=,\cdots,n}$ be a $n\times n$ 0--1 matrix. The ρ-invariant set

$$\sum=\left\{(a_{i})_{i\in Z}\in \mathscr{A}^{Z}| t_{n_{i}n_{i+1}}=1 \text{ were } a_{i}=A_{n_{i}}\right\}$$

is called a subshift of finite tyoe (on symbols $\mathscr{A}$ determined by T). ρ is called a shift transformation.

DEFINITION. A hyperbolic set Λ has a local product structure if there is a positive number δ such that for any $x\in\Lambda$

$$\phi: W_{\delta}^{s}(x:\Lambda)\times W_{\delta}^{u}(x:\Lambda)\longrightarrow\Lambda$$

is a homeomorphism onto a neighbourhood of x in Λ. Here ϕ is given by $\phi(y, z)=W_{2\delta}^{u}(y:\Lambda)\cap W_{2\delta}^{s}(z:\Lambda)$.