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# A relation between the F. and M. Riesz theorem and the structure of LCA groups

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## § 1. Introduction

Let $G$ be a locally compact Abelian group and $M(G)$ the usual Banach algebra of all complex bounded regular measures on $G$.

Let $L^1(G)$ be a set of all functions intergrable on $G$ with respect to the Haar measure $dx$.

Suppose $\Gamma$ is a $LCA$ group. We shall call $\Gamma$ is a topological ordered group or simply ordered group if there exists a closed semigroup $P$ of $\Gamma$ such that (i) $P\cup(-P)=\Gamma$ and (ii) $P\cap(-P)=\{0\}$. Next we shall say that $\Gamma$ is an algebraically ordered group if there exists a semigroup $P$ such that (i) $P\cup(-P)=\Gamma$ and (ii) $P\cap(-P)=\{0\}$. [(1)].

Suppose $\Gamma=\hat{G}$ (the dual group of $G$) is an algebraically ordered group. A measure $\mu\in M(G)$ is said to be of analytic type if $\hat{\mu}(\gamma)=\int_G(-x, \gamma)\, d\mu(x)=0$ for all $\gamma<0$.

We put $M^a(G)=\{\mu\in M(G)\,;\ \mu$ is of analytic type$\}$.

Call a semigroup $S$ satisfies the condition (*) if $S\cup(-S)=\Gamma$ and $S\cap(-S)=\{0\}$.

Our purpose is to prove the following theorem.

THEOREM 1. *Let $G$ be a non-compact LCA group with its dual $\Gamma=\hat{G}$ is algebraically ordered.*

*If* (I) $M^a(G)\subset L^1(G)$

(II) *for any closed subgroup $H$ of $G$ (but $H\neq G$) $M(G/H)$ has a non-zero analytic measure ($H^{\perp}$ (annihilator of $H$) becomes naturally an algebraically ordered group.)*

*then, $G=R$. And moreover $P=(0, \infty)$ or $(-\infty, 0]$, where $P$ is a semigroup which induces algebraically order into $\Gamma$.*

REMARK 1. In the above theorem the condition (II) cannot be weakened. Indeed, let $F\neq\{0\}$ be a compact torsion-free group and $D$ its dual. We put $G=T\oplus D$, then $G$ is a non-compact $LCA$ group, where $T$ is a circle group.

Since $\hat{D}=F$ is an algebraically ordered group, we can construct a semi-