Hokkaido Mathematical Journal Vol. 6 (1977) p. 296-301

Fusion and groups admitting an automorphism of prime order fixing a solvable subgroup

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(Received April 24, 1976)

1. Introduction.

In this paper, we prove the following theorem:

THEOREM 1. *Let G be a finite group. Assume that G admits an automorphism α of order s, s a prime. Assume further that $C_G(\alpha)$ is a (solvable) $\{2, 3, s\}'$-group. Then G is solvable.*

This is a generalization of a theorem of B. Rickman [7], where he took up the case that $C_G(\alpha)$ is a cyclic q-group for some prime $q\geq 5$ distinct from s.

Since $C_G(\alpha)$ is a s'-group, G is a s'-group. Hence, it is well known that $\langle\alpha\rangle$ leaves invariant a Sylow q-subgroup for each prime divisor q of the order of G. Since $C_G(\alpha)$ is of odd order, $C_G(\alpha)$ is solvable by the well-known result of Feit-Thompson [3]. The proof of Theorem 1 depends on the anaysis of the fusion in G. Therefore, we need the following theorems: (For the definitions and notations, see § 2.)

THEOREM 2. *Let G be a finite group and $W_1, \cdots, W_n$ be conjugacy functors for a prime p. Assume that $\{W_1, \cdots, W_n\}$ controls p-fusion in every p-local of G. Then $\{W_1, \cdots, W_n\}$ controls p-fusion in G.*

THEOREM 3. *Let G be a finite group and $W_1, \cdots, W_n$ be conjugacy functors for a prime p such that $W_i(P)\supseteq Z(P)$ for each i and each p-group P. Assume that $\{W_1, \cdots, W_n\}$ controls p-fusion in every p-constrained p-local of G, then $\{W_1, \cdots, W_n, Z\}$ controls p-fusion in G.*

These theorems are generalizations of a well-known theorem of Alperin-Gorenstein [2].

COROLLARY 1. *Let W_1 and W_2 be conjugacy functors satisfying $W_i \supseteq Z$ $(i=1,2)$. Assume that $N=N_N(W_1(P_0))\, N_N(W_2(P_0))\, O_{p'}(N)$ for every p-constrained p-local N of a finite group G and a Sylow p-subgroup P_0 of N. Then $\{W_1, W_2, Z\}$ controls p-fusion in G.*

COROLLARY 2. *Assume that G is S_4-free and $Z(S)\trianglelefteq N_G(J(S))$ for a Sylow 2-subgroup S of G. Then $N_G(Z(S))$ controls 2-fusion in G. In par-*