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On radicals of principal blocks

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§ 1. Introduction

Let K be an algebraically closed field of characteristic p, G a finite group with a p-Sylow subgroup $P\neq 1$, KG the group algebra of G over K and B_1 the principal block of KG with Cartan matrix $C_1=(c_{st})$. Further, we shall represent $[J(KG);K]$ the K-dimension of the radical $J(KG)$ of KG, and u_s, f_s $(s=1,2,\cdots,r)$ the degrees of all principal indecomposable left ideals U_s of KG and all irreducible modules $F_s=U_s/J(U_s)$, respectively, where F_1 is the trivial module.

R. Brauer and C. Nesbitt [1, p. 580] assert $u_1f_s\geq u_s$ for all s and so $[J(KG):K]\leq|G|(1-1/u_1)$. From this estimation, it is easily seen that $[J(KG:K]=|G|(1-1/u_1)$ is equivalent to $u_1f_s=u_s$ for all s. In this paper, we shall call the following question Wallace's problem.

If $[J(KG):K]=|G|(1-1/u_1)$, then is P normal?

As was pointed out by D. A. R. Wallace in Math. Reviews 22 (1961), # 12146, the solution of this problem [8, Theorem] contains an error but holds good for p-solvable groups. Recently, some studies on Wallace's theorem [8, Theorem] are given by Y. Tsushima [7] and the author [5]. The result of R. Brauer and C. Nesbitt [1, p. 580] assert also $[J(B_1):K]\leq[B_1:K]$ $(1-1/u_1)$. And so $[J(B_1):K]=[B_1:K]$ $(1-1/u_1)$ if and only if $u_1f_s=u_s$ for all $F_s\in B_1$.

Using P. Fong's theorem [3, Lemma (3A)], Wallace's theorem [8, Theorem] is slightly modified as the following:

THEOREM A (D. A. R. Wallace). *Let G be a p-solvable group.*

$[J(B_1):K]=[B_1:K](1-1/u_1)$ *if and only if G is a p-solvable group with p-length* 1.

In the present paper, we shall show that if P is cyclic, then $[J(B_1):K]=[B_1:K]$ $(1-1/u_1)$ if and only if G is a p-solvable group with p-length 1. As an immediate consequence of this and Wallace's theorem [8, Theorem], we can see that Wallace's problem is valid for a group with a cyclic p-Sylow subgroup.